

Gödel – Husserl – Platonism

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Abstract

We know that in 1959, Husserl started to replace Leibniz as Gödel's favorite philosopher. We think we know what caused Gödel to shift his theoretical alliance: he perceived a likeness between Husserlian phenomenology and his own goal of demonstrating mathematical realism, and he thought he could use Husserl's phenomenology to find new axioms for set theory. In this essay, we argue for the claim that insofar as their common goal of a robust realism is concerned, it was a marriage made in heaven or, better, it could have been a happy marriage, were only Gödel prepared to accept some qualifications to his Platonic gut intuitions.

Keywords: Gödel, Husserl, Platonism, mathematical objects, realism, idealism, abstract objects

Introduction

Although Platonism has been around since Plato, it is a fairly recent topic in the philosophy of mathematics. In its current meaning, it was first used by Paul Bernays in 1935; Bernays set the meaning of "Platonism" within its contemporary mathematical usage as the postulate advocating the consciousness-independent existence of mathematical objects. He wrote:

Euclid postulates: One can connect two points with a straight line, while Hilbert states the axiom: Given any two points there exists a straight line on which both points lie. Here, "exists" refers to the system of straight lines. This example exhibits already the tendency (which we are talking about) to consider objects as cut loose from all bonds to the thinking subject. This tendency was emphasized in the philosophy of Plato; allow me therefore to call it "Platonism." (Bernays (1935, 53; our translation)¹

What prompted Bernays' remarks were certain developments in late 19th and early 20th century mathematics; a hundred years earlier, in 1835 mathematicians didn't have a Platonism problem.

Traditional mathematics was defined as the science of measure and number (geometry and arithmetic), and both branches were closely tied to the scientific study of the natural world: geometry studied physical space, arithmetic and analysis physical magnitudes and their change over time. Consequently, Jean Le Rond d'Alembert classified mathematics as a "science of nature."² This started to change in the 18th century, when Leonard Euler (followed by Joseph-Louis Lagrange) made analysis more abstract by severing its treatment from the consideration of geometric curves, a development that eventually led to the ε - δ techniques of the 19th century by Augustin-Louis Cauchy and Karl Weierstrass, which in turn triggered a study of the real numbers and which of their properties makes analysis possible (see Gray 2008, chs 2.2, 3.2, and 4.4). The latter had become a pressing issue, since problems in various fields of mathematics, both pure and applied, required the study of classes of functions whose properties (metric, completeness, limits) were like those of the real numbers. During the same time period, investigations into the solvability of equations had led to the development of concepts such as group, ring, or field, which, in the hands of Richard Dedekind and Emmy Noether, led to the formation of modern, abstract algebra (see Kleiner 2007; Gray 2008, chs 2.3, 3.3, and 4.3). Furthermore, since the mid-19th century, counterintuitive conceptions such as non-Euclidean geometries or continuous-but-nowhere-differentiable functions were slowly established as rightful denizens of the mathematical world.³ In short, by the beginning of the 20th century, abstract algebra and set theoretic methods had given rise to new concepts and new fields which, although used in physics (operator theory in quantum physics or non-Euclidean geometry in general relativity theory), could no longer be said to have been abstracted and refined from its study.⁴

Two types of reasoning in particular were contested among mathematicians. First, the law of excluded middle (LEM) (see Troelstra & van Dalen 1988, ch. 1). It says: For any meaningful statement p , either the statement p or its negation

non-p must be true. Controversial was not the LEM itself but its unrestricted applicability to infinite totalities. When presented with a finite collection of objects, we can apply the LEM by checking all objects one by one to see whether they have a certain property—or not. But for an infinite collection, this is no longer possible to do. The significance is this: if we still admit the application of the LEM to an infinite collection, we can prove that objects with specific properties must exist (e.g., that an equation has a solution) *without being able actually to produce these objects*. Other instances of non-constructivity are certain axioms that postulate the existence of equally controversial, infinitary objects (e.g., power set, choice set).

Second, there are impredicative definitions (see Parsons 2002; Crosilla 2017). Let R denote the set of all real numbers. One property of R that emerges as absolutely critical for analysis is their completeness. It can be expressed in various, equivalent ways, but a very versatile formulation is to require that every non-empty subset S that is bounded above has a least one among its upper bounds (the so-called “supremum”). Now, a subset (like S) is not defined unless its superset (like R) is. But we just said that R can only be defined by a property that all suitable subsets S have. This seeming circularity in the definition of R is called its impredicativity; it thus lies at the very foundation and core of modern mathematics. David Hilbert was a staunch defender of the unrestricted use of LEM and of non-constructive and non-predicative definitions, while equally famous contemporaries like Henri Poincaré, Jan Brouwer, or Hermann Weyl had their reservations.

We take it as an assumption that, when it comes to modern mathematics, Kurt Gödel and Edmund Husserl knew what they were talking about. Gödel, as it is well-known, made seminal contributions to and left his mark on modern mathematics (see Dawson 1997). Husserl, a mathematician by training, enjoyed frequent personal interactions with key players in the dispute just sketched (see Schuhmann 1977). Husserl had studied, among others, with Weierstrass in Berlin and completed his doctoral dissertation with Weierstrass’ student Leo Königsberger in Vienna (see Fraser 2019). Later in Halle, when his foreign degree had to be *nostrified* for the

purpose of his habilitation, it was his friend Georg Cantor, the founder of set theory, who examined him.⁵ Furthermore, we now know that he tried to keep current even later in life (see Hartimo 2017b). Hilbert, a preeminent mathematician of his generation, acted, on many occasions, as Husserl's advocate during the latter's time in Göttingen (see Hartimo 2017a and the literature cited) and even suggested, shortly before Husserl retired from the University of Freiburg, that he should return to Göttingen (Vongehr 2013, 17). Weyl, successor of Hilbert in Göttingen, openly adopted Husserlian views in his writings (see Da Silva 2017 and the literature cited). In short, we have reason to take what both Gödel and Husserl say seriously.

We now look at Gödel, Husserl, and Platonism, in that order. Our focus is fairly limited. We do not aim at a comprehensive study of either thinker but instead home in on a single question: After Husserl replaced Leibniz as Gödel's favorite philosopher in 1959 (see Wang 1987, 121), what kind of Platonism, if any, could Gödel have justified based on what he found in Husserl's works regarding the ontological status of mathematical objects?

1. Gödel's Views on the Mathematical Realm

We have a number of excellent, rigorous scholarly studies on our topic we can build upon.⁶ This means, we can be brief and focus on what will be relevant to our discussion. Generally speaking, so that the reader knows where we are coming from, we agree with Parsons—that Gödel's position is characterized by both realism and intuition. (We will be more specific below.) Furthermore, we think that the narrative, as told by van Atten and Kennedy, is convincing (i.e., that Gödel turned to Husserl hoping to find more compelling arguments for his realism) but would give equal weight to Tieszen's observation (i.e., that Gödel hoped to find support for his program of finding new axioms for set theory based on a phenomenological meaning analysis of the concept set).

Gödel was reluctant to publish his philosophical views, because he felt that while his convictions were strong, his arguments were less so.⁷ Another complicating factor was that he avoided clashes of opinions; he preferred being silent over

saying something controversial (see Feferman 1984). Thus, during a time in which he believed himself to represent a minority opinion (“in view of widely held prejudices”), he was hesitant to express his philosophical views at all:

“Of his philosophical interests, it appears that for many years he kept his ideas to himself both because he had not formulated them to his own satisfaction and because he had not found a sufficiently sympathetic audience.” (Wang 1987, 123)

As a result, there are only two published pieces in which Gödel included tentative statements of his views on mathematical existence—Gödel (1990b), his contributions to the Schilpp volume on Russell, and Gödel (1947), a solicited popular exposition of Cantor’s continuum problem, subsequently revised as Gödel (1964). But Gödel left more detailed remarks unpublished. These remarks can be found, among others, in Gödel’s Gibbs-Lecture, his planned contribution to the Schilpp volume on Carnap, or the draft for a lecture probably prepared for a meeting of the American Philosophical Association.⁸ Based on the little he had published, no one seemed to have taken offense at Gödel’s views,⁹ views that Parsons would later call “scandalizing.” (Parsons 1995, 44, 45) It seems fair, therefore, to say that before the publication of his *Collected Works* (1986ff.), Gödel’s views were widely known only thanks to Hao Wang’s book-length studies, which portrayed Gödel as a philosopher.¹⁰ From Wang, we first learned that Gödel considered himself “a conceptual and mathematical realist since about 1925”; that Russell considered him an “unadulterated Platonist”; or that he contributed his breakthrough work as a result of his philosophical views.¹¹

Gödel never arrived at a definite articulation of his views, but we also do not know how much change they underwent, if any. Thus, for the purpose of this paper, we will characterize his views by a number of key features he mentions repeatedly over the course of 20 years (roughly, 1944–1964). We distinguish between passages written before and after 1960, the year after Gödel’s Husserl studies had begun, and we will speak of a “mathematical realm,” so as not to insert any prejudice into the question of whether it is a reality. We collect, for the reader, a sampling of Gödel’s views on the mathematical realm, its

existence, and its existence relative to the physical realm.

A. Pre-1960 quotations

A.1 *A mathematical realm exists*

There is an independent, objective mathematical realm. Independent means it exists “independently of our definitions and constructions” (1990b, 128); it exists “independently of our mental acts and decisions [...³¹¹|³¹²...] our free choice and creative acts” (1995c, 311f.); it is as “independent [...] of our thinking as nature [is]” (2003c, 505). Objective means “we cannot create or change [it] but only perceive and describe [it]” (1995c, 320); its objectivity follows from being “entirely independent of any convention and free choice” (ibid.).

A.2 *What the mathematical realm is*

The mathematical realm consists in the “non-‘tautological’” fabric of “relations between the concepts of mathematics” (1995c, 320) or the system of “properties of those concepts” (1995d, V, 360). These “concepts are composed of primitive ones” (ibid.), and even if their definition is “arbitrary, [...] what can be asserted on [their basis is] objectively determined” (ibid., 359). The mathematical realm is “well-determined” and makes axioms or theorems “either true or false” (1990c, 181).

A.3 *How the mathematical realm and physical realms compare*

The mathematical realm “confronts [...] our thinking as nature [does]” (2003c, 505) and “[its] assumption [...] is quite as legitimate as the assumption of physical bodies and there is quite as much reason to believe in their existence.” (1990b, 137)

A.4 *Mathematical Truth*

Mathematical knowledge is “purely conceptual knowledge” (1990c, 312) and its statements are “analytic” (1995d-III, 347), that is, “true already owing to the meaning of the terms occurring in it [...] that is, the concepts they denote” (1995c, 320).

A.5 *Mathematical knowledge*

Russell’s comparison of “logical evidence with sense perception [...] has been largely justified [...] and will be still more in the future” (1990b, 121); indeed, “the similarity between mathematical intuition and a physical sense is very striking [...] [f]or the difference, as far as it is relevant here, consists solely in the fact that in the first case a relationship between a concept and a

particular object is perceived, while in the second case it is a relationship between concepts” (1995d-V, 359). Mathematical truth can “directly be perceived” (1995d-III, 347) by an “additional sense [... i.e.,] mathematical intuition” (1995d-III, 353), that is, “by means of reason alone” (1995c, 312).

B. Post-1960 statements

B.1 Phenomenology

“[T]here exists today the beginning of a science which claims to possess a systematic method for such clarification of meaning, and that is phenomenology founded by Husserl. [...] But not only is there no objective reason for [its] rejection, but on the contrary one can present reasons in its favor.” (1995e, 383)

B.2 The mathematical realm’s independent existence

“[S]omeone who considers mathematical objects to exist independently of our constructions and of our having an intuition of them individually, and who requires only that the general mathematical concepts must be sufficiently clear for us to be able to recognize their soundness and the truth of the axioms concerning them.” (1990d, 258)

B.3 Knowledge of the mathematical and physical realm

“[D]espite their remoteness from sense experience, we do have something like a perception also of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true. I don’t see any reason why we should have less confidence in this kind of perception, i.e., in mathematical intuition, than in sense perception. [...] It should be noted that mathematical intuition need not be conceived as a faculty giving an immediate knowledge of the objects concerned. Rather it seems that, as in the case of physical experience, we form our ideas also of those objects on the basis of something else which *is* immediately given. [...] Evidently the “given” underlying mathematics is closely related to the abstract elements contained in our empirical ideas.” (1990d, 268)

“[T]he question of the objective existence of the objects of mathematical intuition [...] is an exact replica of the question of the objective existence of the outer world.” (1990d, 268)

Gödel himself once remarked that if a Platonism were ill-defined, then it would not “satisfy any critical mind” (1995b, 50) and was finely attuned to nuances in meaning (see Benacerraf’s observation reported in Moore (1990, 166) and to how much

difference they may make (see Gödel's critical remark (1990b, 127) on Russell's sloppy formulation of the vicious-circle principle). We therefore think it is uncontroversial that Gödel would argue that what specific philosophical position the features we just collected amount to will entirely depend on the way we spell out the meaning of their key terms. This is what we will do in the last section: assign key terms in the quotations above a meaning informed by Husserl's philosophy.

2. Husserl

We should not expect that Husserl never changed his mind during the period of about 50 years (1887–1939) that he published (see Mohanty 1995). Actually, we would consider it a death knell to his philosophizing if he never did. In fact, Husserl's language changed, and it changed enough to cause a rebellion among his students (see Ingarden 1975). But if major changes in Husserl's philosophy had occurred, then they might affect how we understand certain texts. And indeed, there are two obvious candidates for such dramatic changes: the alleged recanting of what Frege called his psychologism with the publication of his *Logical Investigations* in 1900, and the so-called "transcendental turn" he took with publishing the *Ideas* in 1913. In light of recent scholarship (see Centrone 2010 and the literature cited), we believe that the preponderance of evidence suggests that Husserl's mature philosophy is a continuation of tendencies already visible in his earlier philosophy. This means that while Husserl continues to modify his theories, and while new elements are added through the progression of his thought, his responses to various critics do not constitute any radical reversals. We mention this, because it has some bearing on how we interpret certain bold statements that Husserl made in his *Logical Investigations*, which echo the Platonic language of Bolzano's *Theory of Science*.

Given our goal—what could Gödel have found in Husserl?—it makes sense to limit ourselves, wherever possible, to those of Husserl's works that Gödel owned.¹²

2.1 Objectivity: The Bare Account

How to wrestle objectivity from subjective acts, is the one question we see running through Husserl's entire *opus*. The question of objectivity arises within the realm of mathematical objects just because it seems obvious that mathematical entities are the creation of a human mind; to what extent, then, might their existence not be purely subjective? According to Husserl (FTL),

“[people] never had the courage to confront head-on the embarrassing question how subjectivity can create entities that may count as ideal objects of an ideal ‘world.’” (Hua XVII, §100, 267; our translation)¹³

On one hand, it is the individual human being who does mathematics; and they do so within their own personal limitations (say, they are struggling with a proof everyone else finds very easy to understand) or by employing some outstanding ability they may have (e.g., finding a proof for a conjecture that had resisted the attempts of generations). On the other hand, mathematics consists of the lasting products of individual acts of consciousness and how the mathematical community collaborates to systematically forge from these individual contributions an accepted body of mathematical knowledge: fruitful definitions and axioms, communicated or published proofs, papers and theories, and, finally, canonical textbook knowledge.

Where does the objectivity of mathematics come from? Suppose we wander through a museum of modern art and you say: “Oh, look at that fancy chair!” In that situation, we use sense perception to achieve agreement that there is an object in our common field of vision that has certain specific properties. Thus, a shared external object serves as basis for objectivity. But how do we agree that it is a chair? We agree because we both know the meaning of the word “chair.” Thus, a shared meaning serves as basis for objectivity. This much seems uncontroversial.

What is controversial and sets Husserl apart is how he proposed to analyze the two observations. Seeing something and knowing the meaning of a word are mental events, or, in the language of phenomenology (which prefers to remain agnostic with respect to the existence of a “mind”), they are

intentional acts of consciousness. If, following Husserl (see LI V, §§1–8), we identify consciousness with conscious acts, or, to be more precise, with intentional acts,¹⁴ then seeing an object or knowing a word's meaning is an intentional act. Furthermore, according to Husserl, that to which an intentional act is directed is always an object, whether that object is a chair, a word meaning, an isosceles triangle, or a unicorn fantasized under the influence of illicit substances. (An intentional act is only intentional insofar as it takes on an object; that is what intentionality is.) In other words, Husserl wants to identify what constitutes a given intentional act so that it will give rise to objectivity among all who perform it.¹⁵ The mistake of psychologism was not, according to Husserl (see Hua XVII, §56, 160), that it looked at individual psychological acts, but that it failed to notice that consciousness transcends itself into the realm of objectivity.

Thus, objecthood is both the precondition of consciousness and the result of the constituting acts of consciousness. The object is that *of which* consciousness is conscious, without which there would be no intentional act to speak of, and it cannot be reduced to the agreement between various subjects with regard to any particular pre-given object. If we understand “intersubjectivity” to mean general cross-individual agreement and understand “objectivity” to mean independence from any individual subjectivity, then the terms are not synonymous, and neither entails the other. We may agree on something that objectively speaking is not the case (we mistake a beech for a birch) and disagree on something that objectively speaking is the case (we disagree on whether a series converges). More importantly, we would not *yet* be doing phenomenology. These terms acquire much more nuanced definitions over the course of Husserl's thinking (which we will go into later.)

2.2 Phenomenological analysis and epoché

Following Husserl, we want to look at intentional acts; but how do we do that? More often than not, we need the objects we study to appear not how they occur in the wild but primed: appropriately individuated (“cut at their joints”) and

subsequently cleaned, purified, sliced, stained, etc. The first step to phenomenological analysis is to objectivate the intentional act—to take it as the object for analysis. We have to consider a particular intentional act as if it were removed from the entire stream of consciousness, though it is immediately apparent that this is not how an intentional act is experienced. (It is, in actuality, part of the stream of consciousness; to remove it is to consider it as it does not appear in the wild.) First, we need a sound taxonomy of intentional acts; then we need to learn how to prime them.¹⁶ While for our purposes we can ignore questions of taxonomy, we have to mention priming. Phenomenological priming is not required of the object but of the observer; for the main risk is for the researcher to contaminate the specimen.

The main contamination risk is caused by what Husserl calls the natural attitude; that is, the attitude of our everyday lives.¹⁷ The natural attitude comes with many beliefs—for instance, that the tree I see really exists—that outdazzle and hence hinder a sober analysis of all the finer nuances of what is or is not actually given in consciousness. And already supposing that there are extra-mental objects (objects that transcend consciousness) that then enter my consciousness, is an assumption we cannot make. Whether we have (or do not have) a license to make this assumption could, however, be an outcome of the phenomenological analysis at a much later stage; initially, it is a complete no-go. We have to set aside considerations of the modes of existence of the objects of consciousness, which are to fall out of our analysis at some eventual stage. The suppression of the natural attitude is what Husserl calls “bracketing” or the *epoché*. It is the first in a sequence of steps, called the phenomenological reduction, which are meant to support an unobstructed and uncontaminated view of what is given to consciousness. We do not specifically *doubt* the existence of the world, when we bracket it; we put it in parentheses, for the moment. When I perform the *epoché*,

“[it] shuts me off, *eo ipso*, from effecting any judgment, from taking any position predicatively toward being and being-thus and all the modalities of being which pertain to the spatiotemporal factual being of anything ‘real’.” (Husserl 1982, §27, 51)

That is not to say that we deny the reality of the universe. We set it aside, or bracket it off. In what follows, we just need the first step, the *epoché*, and will bracket things as needed.

An immediate and legitimate worry arises: for any phenomenological analysis, it seems to belong to a pre-scientific period of predominantly introspective psychology,¹⁸ or, as Auguste Comte argued, is outright self-contradictory:

We can indeed note that by some indomitable necessity the human mind can directly observe all phenomena, except one's own. For by whom should the observation be done? The thinking individual cannot split itself into two, one of which does the thinking while the other observes the thinking. How could an observation occur when, as in this case, the observed organ and the observing organ are identical? (Comte 1869, 30ff.; our translation)¹⁹

Here we assume that such Comtean concerns have been successfully alleviated (see Zahavi 2005); in fact, Husserl never saw introspection as a possible contradiction just for the fact that it happens all the time. (Only the least philosophical philosopher denies what is immediately apparent for the sake of making some technically attractive but altogether false claim.) Still, we would like to take this as an opportunity to clearly state that we take any phenomenological analysis to be as fallible as any other human endeavor.²⁰ We further believe that a phenomenological analysis need not be autonomous but may benefit and take cues from the neurosciences, nor is it immune from WEIRD distortions.²¹ In short, we do not assume phenomenological analysis to enjoy any special first-person privileges familiar from a Cartesian philosophy of mind.²²

With consciousness' capacity to be conscious of itself, among other objects, we can properly begin an analysis of intentional acts and their objects.

2.3 Intentional acts and their objects

Conscious acts are intentional insofar they are about something. For instance, I see *X*, or know *X*, or detest *X*, etc. The mark of the conscious is its intentionality, this *being about some X*. When we discuss an intentional act that is about some *X*, we call *X* the intentional object. This suggests a view prevalent in the philosophy of mind but with which Husserl

would vehemently disagree—that we have, on one hand, intentional acts and, on the other hand, the intentional objects, and both as separate entities. This split is a familiar theme; we speak of propositional attitudes and propositions and assume that propositions (e.g., as Fregean thoughts) are entities that exist independently from any attitudes. This is wrong, however, in phenomenology (see Hua XIX, II.1 and V, §11).

An intentional act of our consciousness is never an empty act, like a container, which is then filled with an object (objectual reading). Likewise, it is not quite correct to say that the intentional act is *modified* by an object, like my sense of temperature is when I feel heat near a fireplace (adverbial reading). But to say that an act of consciousness is modified by its object is like saying a pain is *modified* by the accidental striking of a clumsy shin on a misplaced coffee table—not quite correct (because it implies that the pain exists prior to the strike, or that consciousness can exist that isn't consciousness of anything). It is difficult to use a SVP language to express the matter more correctly when employing the verb-phrase (adverbial reading) or the predicate phrase (objectual reading) fall short of the task. Can we put it into the subject? It might help to picture consciousness as a shapeshifting slime whose outer shape constantly undergoes change; then we can call its temporary stages intentional acts and their outer shapes their objects, but we would still fall short. However poor the analogy, it makes clear the Aristotelean hylomorphism that enters the phenomenological analysis. There are forms of consciousness that are instantiated in particular intentional acts, of which the material is their content—that of which we are conscious, i.e., the object. Were there no object of consciousness, there would be no act of consciousness. (The slime would not exist.) Consciousness is dependent on (or relative to) its object. Consciousness is always consciousness of something. That is to say, to speak of “consciousness” as if it were something independent is already to have abstracted it from how it appears to us. In the wild, the object of consciousness is not separate from the intentional act; we may consider it as separate, though it is not. The fact that we can consider consciousness and its object as separately existing makes it

seem as if consciousness is an independently existing *thing*, a subject or predicate, when in fact the object is so integral to the act of consciousness that *without the world* for consciousness to be conscious of, there would be no consciousness.

In order to emphasize this point, it would be good to say something like: an intentional object is woven into the fabric of its intentional act like so many threads. But then language gets convoluted quickly. So when we say an object is given in an act, or that the object is woven into the fabric of consciousness, this is what we mean (i.e., that the object of consciousness is the content of an act of consciousness like thread is the content of a fabric, where in this analogy, the fabric is an intentional act.)

2.4 Fixing our language: concrete and ideal, real, signify and transcendent

Sometimes we feel we understand something better when we translate into a language that is more familiar to us; this is not the goal of the present section. This section is merely to fix our language without any claim as to explanatory power. Note in particular that the terminology we are going to introduce is meant to be neutral in respect to the natural attitude and whether things really exist or not (whatever that means).

If we look at a tree, then the tree will be given to our consciousness by what Husserl calls *sensible intuition*. If we assume a metric space, then the space will be given to our consciousness by our conceptual grasp of its definition and the concepts involved. Husserl has a fancy word for conceptual grasp (or for comprehending the meaning of words): categorical or *eidetic intuition*. Following Husserl, we call objects given to sensible intuition *concrete objects*, while objects given to eidetic intuition we call *ideal objects*. Thus, given to consciousness, a tree is a concrete object, while the number π or the meaning of the word “chair” is an ideal object. (The distinction concrete/ideal resembles the distinction we make in plain English when we speak of physical and abstract objects, and often indeed Husserl differentiates the two according to the fact that the former is spatio-temporal.)

An intentional object is given to a consciousness as woven into the fabric of its act; this alone does not allow us to go

beyond its presence to a consciousness. (We say “present to” and not “present in” to avoid the misleading associations of the container theory of consciousness.) While we have a natural impulse to jump to conclusions about the reality of certain objects (whether they exist in the real world or are mere hallucinations, say, caused by high fever), all we have to start is that they are present to our consciousness. In the words of Husserl:

It makes no essential difference to an object presented and given to consciousness whether it exists, or is fictitious, or is perhaps completely absurd. (Husserl 1970, vol. II, 99; cf. Hua XIX, V, § 11)

Note that, insofar both ideal objects and concrete objects are present to a consciousness, they share the same ontological status of “being present to a consciousness.” Thus, the tree I see in the back yard and the number π I use in an equation share the same ontological status. (And that therefore, if concrete objects are real, so are *ideal objects*.) This is one point where we can identify a platonic bent to Husserl’s phenomenology, but as we argue, misguided, for Husserl is saying that ideal objects are as real as concrete ones, whereas Plato would never say such a thing.

According to Husserl, intentional objects *signify*. To signify in consciousness is what to denote is for language: to point at something beyond (i.e., consciousness or language). But, while we are engaged in *epoché*, there is no a priori ground for assuming that what an intentional object signifies—say, a real tree in the real world, or the true metric space in some Platonic realm—exists somehow independently of consciousness. We call what is signified (e.g., the alleged real tree) a *transcendent object*. And we call it so, since its putative existence goes beyond, or transcends what is given to a consciousness. We thus see that a concrete object in consciousness serves as a sign for a transcendent object. We say that the transcendent object is what is *meant* by the concrete object, which is to say that the intentional object (i.e., a presentation) is a sign that directs us towards the existence of some object *not* of consciousness (i.e., a thing). We can conceive of ideal objects as signs, too, but what they signify may not be beyond consciousness.

Note the radical claim we sneaked in above when we

said that comprehending word meaning is an intentional act (eidetic intuition) best described in analogy to a perceptual act (sensible intuition). This runs counter to the traditional divide between, on one hand, intuition, which passively receives but does not comprehend, and, on the other hand, understanding, which is active and does comprehend. Husserl disagrees; perception is also an activity, one of the many possible forms of consciousness. According to a naive epistemology, we are given something in intuition (patch of green) and then our understanding forms a judgement (it is a tree). According to Kant, what appears as a raw given in intuition is already constituted by a rational mind (greenish substance causally connected in space-time); this then allows understanding and judgement to find traction (Kant's schematism). According to Husserl, who tries to correct and extend Kant's analysis, everything, whether concrete or ideal, is first intuited in an intentional act whose fabric is much more complex than Kant assumed. The more accurate the phenomenological analysis, the more details we can bring out of what is given in the intentional act "tree in the back yard" or "let x be a set." Note that the analysis of eidetic intuition may go beyond a mere conceptual analysis, the familiar analysis of meaning as the hallmark of philosophic methodology since Plato's *dihairesis*. The phenomenological analysis of the full conscious experience will discern many different layers, among them layers of body awareness and situatedness in the world. For, according to Husserl, consciousness is embodied.²³ This richness of intentional acts will prove critical for Husserl's transcendental constitution of intersubjectivity. Recall, however, that we do not claim any special first-order privileges: the richness of the given may not be immediate, and it may take time and training to bring it out.

2.5 Aspects and full/partial presentations

Suppose we look at a tree from different sides. You see things I do not see, and vice versa. Is it a different tree, because we ascribe it different properties? No, we assume it to be the exact same, identical tree and our differently perceived properties to be complementary. Suppose we both say "number

π,” but you think of the ratio of a circle’s circumference and diameter: $\pi = C/d$, while I think of the arc length of the top half of the unit circle:

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}},$$

or some infinite series:

$$\sum_{k=1}^{\infty} (-1)^{k+1} \frac{4}{2k-1} = \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \frac{4}{13} - \dots$$

Is this a different number? No, we assume it to be the exact same, identical number and our different descriptions to be complementary. (Frege’s distinction according to sense and meaning comes to mind here.)

Generalizing the observation above, namely, that we may miss something based on our perspective in space, we say that an intentional object may have many different aspects, not all of which must be fulfilled in a single presentation to consciousness. If all its aspects are fulfilled, we call it a *full* presentation; otherwise, a *partial* (i.e., a partially fulfilled) presentation.

For instance, the die in front of me has six sides, but I see only three. Thus, the intentional object “die” given in sensible intuition has unfulfilled aspects, namely, the three sides I cannot see, and is therefore partially presented. Likewise, the number π, given in eidetic intuition, will have many unfulfilled aspects, namely, all those representations of it as an infinite series that I am not aware of (but Euler was). It makes sense, therefore, to speak of fulfilled and unfulfilled, or partially fulfilled, presentations.

The same observations we just made apply to the situation where we speak not of two presentations of the same intentional object but speak of a concrete object and the transcendent object it signifies. They are the same, but they may be presented in different aspects to my consciousness. Because you and I are faced with different presentations does not mean that there are two distinct objects, one present to each of us as conscious subjects. The presentation of an object does not duplicate it, even if the object itself maintains unfulfilled aspects. As Husserl put it:

It need only be said that the intentional object of a presentation is *the same* as its actual object, and on occasion as its external object, and that it is absurd to distinguish between them. (Husserl 1970, vol. II, 127)²⁴

It is a fallacy of the natural attitude to reduplicate the intentional object and split into what is given and into what it signifies; the impulse to do so is strong but, in light of a proper *epoché*, still misguided.

We can now say how concrete and ideal objects differ. If the signified object is a concrete object, a partially fulfilled presentation of its aspects is inevitable. Concrete objects are always presented in space, and this is a simple reason for their necessarily partial presentation; for spatial objects will always have sides I cannot see. If, however, the signified object is an ideal object, then the degree of fulfillment of its presentation will depend on my grasp of its conceptual complexity. We might never arrive at a full presentation of what are usually called empirical concepts (such as tree or chair), but full presentation seems like a viable option for mathematical concepts (e.g., by definition up to isomorphism).

The proper application of the *epoché* does, however, justify our distinguishing between two types of objects. The two modes of presentation indicate two types of objects, be they only objects of consciousness. We argue that this does constitute an ontological difference.

2.6 An ontological difference

We may want to use the difference in fulfillment to distinguish the ontological status of what concrete and ideal objects signify. (And we can use it instead of or in addition to spatial presentation.) On one hand, there is the concrete object. Its inevitably partially fulfilled presentation to consciousness makes our natural attitude assume that the transcendent object, which it signifies, is external to consciousness. On the other hand, there is the ideal object. Its, at least in principle, possibly full presentation to consciousness makes our natural attitude assume that it is identical to what it signifies and hence, internal to consciousness (that ideal objects are subjective, the original problem Husserl and Gödel were

facing).

We said earlier (in 2.4) that ideal objects and concrete objects have a comparable ontological status, namely, the status of being real, *provided* they are both considered as intentional objects of a consciousness which is itself real. We further said that all intentional objects are signs. What they are signs *of* does not necessarily enjoy comparable ontological status. To put it differently, all objects of consciousness, considered as such, are real. What they signify, however, can be either outside or inside consciousness. (Note. If that were our topic, we could go from here and recover features of the natural attitude we bracketed earlier.)

To think of something is therefore not to make its transcendent object external to consciousness, i.e., we cannot conclude that because we can think an ideal object, that it therefore exists outside of consciousness, perhaps in and amongst the spatio-temporal objects (see 3.2 below). For example, while we say that consciousness is consciousness *of*, it could be conscious of itself. In that case, consciousness is conscious of something ideal. Another example is unity, which, for Husserl, is a form of consciousness but also the basis of our numerical system. It is real, because consciousness is real. But that does not mean it is external to consciousness. The same applies to Dedekind's proof that an infinite sets exist (see Dedekind 1888, 357: Theorem 66); what he meant, in light of Husserl's analysis, is that such an infinite set is real as an ideal object. Husserl mentions Anselm's ontological argument as an example of how philosophy has, at several points, been mistaken in equating an ideal object with an external object.²⁵

With that being said—that the real existence of ideal objects that transcend the intentional acts that signify them does not indicate a consciousness-independent existence—we can answer half of our questions about Platonism. To answer them all, we need to take two more steps.

2.7. Intersubjectivity

Husserl spilled much ink on the topic of intersubjectivity, esp. after his so-called transcendental turn, but little did he include to his published writings.²⁶ Kant could

adduce the validity of Aristotelean logic, which was considered a given, to argue for the intersubjectivity and objectivity of knowledge. Husserl of the earlier, realist phase could have appealed to a shared human condition and, consequently, a shared form of consciousness that would guarantee intersubjectivity and objectivity. But once we commit to an *epoché*, pretty little is left to build on. It is no surprise, then, that Husserl clearly saw the specter of a “transcendental solipsism” (Hua I, V, §42, 91) looming as a threat over his entire phenomenological project.²⁷ Husserl’s argument is long; it spans the entire Fifth of his *Cartesian Meditations* (Hua I, 121–177). We report here only on the main steps.

Assume we practice radical *epoché*, bracket everything and especially what we came to call the “intentional stance.” (see Dennett 1987, ch. 2) We are left with a primordial world that, although purged of all traces of alien subjectivity, still forms a coherent layer of the *Weltphänomen* present to my consciousness (i.e., a “phenomenon of a world” without pretense to its existence) (Hua I, §44, 127)²⁸. In this remaining layer of “mere nature” I find “my own body as the unique one that is not just a body but the only object to which I empirically ascribe fields of sensations.” (Hua I, §44, 128; our translation)²⁹ From here, the primordial world, Husserl proceeds in three steps: First, the constitution of other conscious agents; second, the ontological promotion of my primordial world to that of a world I share with other conscious agents; third, intersubjectivity, i.e., the constitution of a world all conscious agents share.³⁰

Husserl distinguishes between perception and apperception (but different than Leibniz or Kant did). When I see a hammer, its front side is what I perceive. But more is present to my consciousness than what I see; e.g., I know that it forms a solid in three-dimensional space and therefore has a backside. This extra is what I apperceive. In phenomenology, it is “schematized”. While this much may sound familiar, Husserl goes on step further and explains how natural kind terms come about. There was a first time I saw someone using a hammer, say, for driving a nail into a wall. Rooted in this first experience—which added meaning to what was initially presented to consciousness—every time I now see a hammer, I

anticipate its use for driving nails into a wall. Husserl calls this kind of analogical apperception, which is based on forming matching pairs between a past and a present experience, an appresentation (because additional meaning gets co-presented to consciousness; see Hua I, §§ 50ff.). He then argues that in the primordial world, where my body is the only seat of subjectivity, when I encounter another human body that looks and moves like mine, both form a matching pair—the role of empathy!—and I immediately appresent the subjectivity in the other body that I know to inhabit my own. This completes Husserl’s first step: appresentation populates my primordial world with other conscious human agents.³¹

Once these other consciousnesses are given, they change the ontological status of transcendent objects. The way appresentation was defined, I can appresent any other human consciousness only in likeness to my own. This entails that when a concrete object is present to my consciousness, I appresent that it is potentially present to other human consciousnesses as well. In other words, any concrete object is now apperceived as being a potential concrete object for any other human agent. (We say “potential” since it will depend, among others, on the relative position of the other body to my own.) As Husserl puts it:

“The ontological meaning of world, and of objective nature in particular, includes this ‘*thereness-for-everyone*’ that we always appresent.” (Hua I, §43, 124; our translation)³²

Suppose, by some freak accident of nature, someone living a fairly normal life on a remote island without having ever encountered an animal body. When they see a tree, it would miss the appresentive quality of “it’s-there-for-everyone-to-see” which is so absolutely fundamental to our human experience. Thus, the appresentation of the other human body as the seat of a consciousness turns my subjective world into an intersubjective one; here, “intersubjective” has still the limited meaning of “my subjective world is shared with other human consciousnesses.”

In a third step, I realize that the “thereness-for-everyone” that I learned to apperceive with every concrete object is likewise presented to any other human consciousness.

For I conceive of those only as likenesses of my own. Thus, the world I first appresented as shared with other conscious agents, I now appresent as the world they appresent as shared with me and everyone else. We thus arrive at the constitution of an intersubjective world in its full meaning: my subjective world is the one I share with others, and they share theirs with me and everyone else. Moreover, the claim that it is the same world we share and hence can agree on—rather than everyone living in their own bubble—follows from the basic fact of appresentation: I can conceive of any other human consciousness only in likeness to my own.³³

While full intersubjectivity is a notion we can put to good use in Section 3.3 below, it is not yet objectivity as defined in 2.1 above. To this task we turn in the next section, where we recover a notion of objectivity as independence from any individual subjectivity; but do so only in respect to mathematics.

2.8 Objectivity: The Fuller Account

In the *Cartesian Meditations*, Husserl makes already a number of remarks about cultural artifacts (e.g., books, tools, works of art, etc.)—“objects with spiritual predicates” he calls them, since their presence points to alien subjectivity as their origin—and states that they, too, have the objectivity of “thereness-for-everyone” relative to a shared cultural background (e.g., European or French).³⁴ As humans, we are denizens of both the natural world and a cultural world; moreover, we experience both as objective: the objectivity of the former is unconditional (since the natural world stays the same across cultures) while the latter is conditional (i.e., relative to being a member of a certain culture) (see Hua I, §58, 159–163, *passim*). These ideas are further developed in the manuscript *On the Origins of Geometry*.

Some cultural artifacts such as paintings or architecture are created in a medium that automatically grants them a certain amount of permanence; but others are of a more fleeting nature: a tune, a poem, a prayer. They are conceived in a single human consciousness where they can be retained in memory and, if all goes well, recreated; but they will perish unless they

get transferred to a more permanent medium (see Husserl 1976a, 370). Language, according to Husserl, is such a medium; language provides the fleeting creations of a human consciousness with a lasting physical basis: they are grafted onto the body of a language and thus inherit the latter's permanence.³⁵ They last as long as the language is spoken or its records are understood. While these fleeting creations are, in Husserl's terminology, ideal objects, they thus receive a concrete avatar: be it the artist's brain or a book that preserves them for the next generation. Tradition forms when successive generations pass down what they have preserved.

Fleeting cultural artifacts acquire objectivity through tradition for two reasons. First,³⁶ a written record allows other members of the community to recreate the original creative idea or insight; repetition reinforced retention in the first consciousness, and frequent repetition by others consolidates its permanent place within and keeps it a living tradition (see Husserl 1976a, 371.18–378.7 in particular). And this opportunity to recreate constitutes, according to Husserl (see *ibid.*, 367.44), *thereness-for-everyone*, his favorite hallmark of objectivity. Furthermore, since we are dealing with ideal objects, when we recreate, we create tokens of the same type—we sing the same national anthem every time we do—and do not add copies as we do when we recreate concrete objects (see *ibid.*, 368). The second reason is a constant unspoken companion but made explicit in appendices to the *Crisis*; in short, it is the idea that tradition generates a historical *a priori*. Coming out of history, it is contingent; but for those born into it, it is experienced as *a priori*.

“What about the objectivity of these ideal entities? What about this *a priori* of history and its objectivity? We are thus lead again to the precondition of a non-interrupted tradition.” (1976b, 362)³⁷

“The whole cultural present ‘implies’ the whole cultural past. To be more precise, it implies a gapless sequence of cultural pasts that imply one another: the universal *a priori* of history.” (1976a, 379)³⁸

Cultural objects, appropriated by generations of speakers, are experienced like objects of nature: something that is *there-for-everyone* and whose existence is independent of one's individual making. We all can read Shakespeare, and as individuals

Beethoven's Ninth is as much beyond our control as the planet Venus is. Husserl is clear, though, that culture must not be monolithic³⁹: a currency is for all, Kant probably less so.

Husserl conceives of mathematical objects (definitions, theorems, proofs, techniques for calculation) as cultural artifacts. They originate in a single human consciousness, get shared, and eventually become part of the oral or written mathematical subculture (or, to a certain extent, part of the general culture). As such, we encounter them as objective: Euclid is there for everyone, and his theorems are facts beyond my control. New mathematics originates in an individual consciousness; but once it has become part of the folklore or textbook knowledge, it is experienced as independent of my individual subjectivity: it is part of the mathematical tradition that is in place whether I know or think about it or not.

Is objectivity, naturalized by a historical a priori, incompatible with Husserl's earlier, more boldly Platonic language? If we look the program underlying the *Crisis*, namely,

“We can obtain a seriously scientific foundation of our a priori sciences only by an appeal to this a priori [of the life world] we have to develop,” (Hua VI, §36, 144; our translation)⁴⁰

then it becomes clear that Husserl is not going back on what he said, but that he rather admits to his own former oversight and tries to remedy it:

“The supposedly totally independent logic qua universal basic science a priori is nothing but a naïveté. Its evidence lacks the scientific justification from the universal a priori of the life world.” (ibid., 144; our translation)⁴¹

Consequently, Husserl's language in the *Crisis* is not changed much; to wit, he freely invokes Bolzano's terminology of “as-such” (ibid. §34.e, 132) or writes:

Our apodictic [mathematical] thinking—progressing in stages according to concepts, theorems, reasoning, and proofs towards infinity—‘discovers’ only what in truth is already there. (ibid., §8, 19)

If we limit ourselves here to the case of mathematics (and not try address the bigger issue of phenomenology and history)

then what we see Husserl emphasizing in *Crisis* is that mathematical concepts are obtained by a process of idealization. But once we have them, it is business as usual: we explore the logical edifice they form as an extended Leibnizian *mathesis universalis* (ibid., 44).

3. Platonism

As we said above, Platonism has existed much longer than it has been a problem for mathematics. And the definition of “Platonism” Bernays had in mind when he applied the term to mathematics does not necessarily accord with any Platonism Plato might have held. In order to figure out how Gödel might have thought Husserl could support his Platonic conception of mathematics, we first distinguish, in a very coarse manner, between types of Platonism.

3.1 Platonic Platonism

We (the philosophical community) generally think that “Platonism” indicates one or more beliefs that Plato held, subject to interpretation of course. For example, “Platonism” entails a theory of forms, the participation of sensible objects in those forms, and the assumed inferiority of everything sensible since, on Plato’s account, every sensible object, i.e., the entirety of nature, is a shoddy replica of its ideal form. On Plato’s account, mathematical objects are granted a reality above the sensible but below the forms and are accessed through a distinct kind of thought (*dianoia*). Plato explains in the *Republic* Book VI, as part of his exposition of the divided line, that mathematical objects are a subset of intelligible objects, but subordinate to the intelligible objects of which we become aware through the process of dialectic (i.e., the intelligible objects accessible to *nous*). Mathematical objects are not sensible objects, but are assumptions illustrated through sensible objects, as when a geometer draws a triangle, all the while understanding that the proper object of analysis is not the drawn triangle but the ideal triangle that the drawn triangle exemplifies. The intelligible triangle is *assumed* but not clarified. (Thus, it appears to *dianoia* rather than *nous*.) The understanding of mathematical objects is mediated by

sensible experience, unlike the intelligible objects appearing through rationality alone.⁴² None of this is part of what we find when we talk about Husserl's or Gödel's views. Gödel's view is actually contrary to Plato's Platonism, on the point that Gödel's takes sensible reality as the standard against which the reality of ideal objects is to be measured. Likewise for Husserl, whose phenomenology has no room for forms and their shoddy replicas. So if we should find cause to call Husserl, or Gödel, or maybe both, a Platonist, then they are not Platonic Platonists.

3.2 *Lost Island Platonism*

A bold version of mathematical Platonism is what we may want to call "Lost Island Platonism." Go back in time before Google Earth and before airplanes. Think of an island that no one has ever set foot on because it was never spotted from a sailing boat. It exists, although no one has discovered it; and if no one does, it remains forever a lost island. Once you happen to discover it, however, you can take full possession of its treasures. Are mathematical objects like a lost island waiting to be discovered; and if no one does, do they still exist?

Husserl, for one, is not a "Lost Island Platonist." He may seem to be one, however, especially in Book 1 of the *Logical Investigations*; for instance, when he writes in his critique of the anthropologism he found, among others, in Christoph Sigwart:

"The validity of these [logical] laws does not depend on whether we, or anyone else, is able actually to perform acts of conceptual grasp."

"What is true is absolute, is true 'as such,' truth is identically one whether humans or non-humans, angles or gods grasp it in their judgements."

"But every truth as such remains what it is, it keeps its ideal being. It belongs to the realm of what is absolutely valid." (Hua XVIII, I, §29, 109; our translation)⁴³

We first note that Husserl does not speak about the existence, be it of concrete or ideal objects, but about validity (*Geltung*). He explicitly rejects the proposal to translate the "ideal being" of validity as a being in some Platonic realm ("to hang somewhere in the void" (*irgendwo im Leeren*, tr. J.H. Findlay),

p. 136) but insists that “we experience [the idea of truth] like any other idea in act of ideation based on an intuition;” i.e., ideas are woven into the fabric of intentional acts. We note, second, that this denial of a Platonic realm is consistent with his three-step program for a pure logic, or *mathesis universalis*, that he sketches at the end of Book 1 of the *Logical Investigations*:

(1) The identification and systematization of all primitive concepts which, to be clear, “can only originate in respect to the diverse functions of thinking, can only have their foundation in possible acts of thinking” (Hua XVIII, I, §67, 245)⁴⁴

(2) Theoretical laws that hold for these primitive concepts and form a unified theory. These laws have “objective validity” and are “directly rooted in the primitive concepts.” (ibid., §68, 247)⁴⁵

(3) A theory of manifolds “whose deduction rest entirely in those theories [formed in the second step].” (ibid., §69, 249)⁴⁶

Here, we see Husserl making the same point: the entire project of a *mathesis universalis* rests ultimately in the acts of thinking agents. If, then, there is no room for a Platonic realm, where does validity live? It lives in the potential of what thinking agents can do:

“The being, or the validity, of what holds generally is the same as ideal possibility. The statements ‘the truth is valid’ and ‘thinking beings are conceivable that comprehend the relevant meaning’ are equivalent.” (Hua XVIII, I, §39, 135)⁴⁷

Thus, validity does not translate into a claim about the existence of objects but about the possibility of actions. If you give me two Lego bricks, I can snap them together top to bottom; this is a concrete possibility. If you give me two premises (e.g., $a < b$ and $b < c$), I can infer what they entail (i.e., $a < c$); this is an ideal possibility. We therefore find that even Husserl’s bold Platonic language in the first book of the *Logical Investigations* does not, on a closer reading, support a Platonist ontology.

Earlier in his career, Husserl took it for granted that concepts have nice properties suitable for general laws and theories a priori (that, so to speak, Lego bricks have studs). Properties, by the way, that are given in intentional acts and are as such objective because not under our control. Where do

these properties, where does the a priori come from? This is what we see him addressing in his last writings when he includes the life-world and the historical a priori (see 2.7 above). As Johanna Tito (1990, xlvi) put it so nicely:

“Contrary to Plato, for whom ideas are known prior to life, for Husserl ideas are lived before they are known.”

3.3 Husserlian Responses to Gödelian Worries

In a first response to Gödel, we connect his requirements for a satisfactory philosophy of mathematics (collected earlier, see 1.A+B above) and pair them up with various observations from Husserl.

Gödel wanted to argue that the *mathematical realm consists* in the “well-determined” fabric of “relations between concepts” that makes all axioms and theorems “either true or false.” He wanted to argue that mathematical knowledge is “purely conceptual” and “true owing to the meaning of the terms.” The project of a *mathesis universalis* (see 3.2 above) so closely resembles Gödel’s views on this that it is easy to find matching quotations in Husserl for each of Gödel’s claims.

Gödel also wanted to argue that *mathematical concepts* and the relations among them exist *objectively*. There are three senses in which mathematics can be said to be objective according to Husserl. First (see 2.8), the basic concepts are originally given in the intentional acts of the first mathematician. As such, as something given, they are experienced as objective. Any property they have cannot be changed, but described (ideation) and genetically explained (life-world). Second (see 2.8), once they have become part of mathematics, we can either recreate the original experience of the first mathematician (keeping the tradition alive) or encounter them as dead cultural artifacts; either way, we experience their ontological status as something objective. Third (see 3.2), it is a brute fact that certain relations among concepts just hold. Any sufficiently prepared being can find this out. This, their validity, is objective, too. And it does not go away, even if the human race falls out of existence. For in Husserl, validity translates into possibility and not into an

object (that then would have to reside in some location, be it in a consciousness or “somewhere in the void”).

Gödel aimed to demonstrate that the mathematical realm “confronts our thinking as nature” does. If we understand this as the requirement that physical and abstract objects share the same ontological status, then they do so since concrete and ideal objects are both real (in the sense of 2.4 and 2.6). If we understand this as the requirement of a “second plane of reality,” then it still comes out as true. For we encounter nature as intersubjective (thereness-for-everyone) and objective (no single consciousness can change it at will). But this is how we encounter the cultural word: as “harsh realities” (*harte Wirklichkeiten*) (Hua IV, §152, 354). Therefore, since mathematics is an integral part of our cultural word, we encounter it the same (see 2.7–8 above). The mathematical realm loses, however, any metaphysical nimbus; it becomes part of our mundane culture. It is, if you will, a phenomenological version of semantic externalism (see Putnam 1975).

Gödel believed that mathematical truths could be “directly perceived” by an “additional sense of mathematical intuition” which is “strikingly similar to the “physical sense.” According to Husserl (see above) everything, whether concrete or ideal, is first intuited, in case of mathematics in eidetic intuition (see **mo**). We may intuit things blurry at first and need closer inspection to intuit them sufficiently sharply. Eidetic intuition is not really an additional sense, though. It is a faculty we employ all the time; false epistemology just made us overlook this.

Some may object that what we just proposed is cheating; we foisted a position on Gödel that is a Platonism by the letter but not in spirit. Maybe so; maybe Gödel was hoping for a more traditional account, where Platonists exercise divine thoughts in heavenly abodes. But we believe that he kept studying Husserl for a reason, and we believe the reason was that he was willing to reconsider a reinterpretation of his initial Platonic hunches and to get educated on how to spell them out in a more defensible way.⁴⁸ If this is correct, then his education would have included another twist that Husserl brings to Gödel’s

worries, an almost complete reversal of assumptions a Cartesian common sense makes.

Gödel argues that mathematical objects “confront us as objectively and independently of our thinking.” Gödel thus places mathematical objects among the objective as opposed to the subjective, which is to say that he believes in independently existing mathematical objects that would not fall out of existence along with the conscious entity that is conscious of it. The subjectivist, however, would place mathematical entities in the consciousness of the human subject. If we apply the subjective-objective dichotomy to theories of philosophical idealism then Husserl’s phenomenology clearly falls on the side of objective idealism: there are ideal objects, and they are objective—but objective in the senses above, as opposed to any colloquial sense.

One of the fundamentals principles of phenomenology is, however (think *epoché*), not to assume too much. In particular, do not to assume we know what the subject is, and then to define the object relatively to consciousness. The familiar tenet “consciousness is always consciousness of” defines the object insofar as the object is that of which consciousness is conscious of—whatever that may be. If consciousness is conscious of something, then that something is *objectivated*, and now it is an object. The distinction according to subject and object is, then, according to Husserl, not a pre-given primitive fact we are forced to accept as our starting point. Rather, the primitive fact is consciousness and the subjective-objective distinction is derived from the relative role both take in a particular intentional act. The terms “subject” and “object” are defined secondarily only and relatively to the act of consciousness. To say, therefore, that ideal objects exist independently of consciousness is already to assume the subject–object distinction that phenomenology has already denied. Along with the subject–object distinction, or, more technically speaking, with the specification that subjects and objects are defined relatively according to what role they take in a particular intentional act, the ideas of something’s being “internal” and “external” to consciousness disappear as well.

A similar argument can be made to refute dualism. The

dualist conception of reality led us to believe in two distinct substances (the mind and body) and then to try to define their interaction, as if they were two things completely distinct from one another, with a possibly unbridgeable gap in between. But the fact that *consciousness is always consciousness of* means that *consciousness* is not an independent existent, nor probably even a complete existent (in the sense of something whole); it depends on an object (a world of objects), without which, there would be no such thing as consciousness. That is to say, the world is assumed in the definition of consciousness, and consciousness is nothing separate or independent from the world (“world” here meant to indicate the whole bevy of both sensible and ideal objects or, in fact, any object whatsoever), and in fact depends on it for its existence. More properly speaking, it is not accurate to say that anything exists “in” a subject, as “subject” is just a word we use to indicate whatever it is that is conscious *of* the world (as a fabric is a fabric of threads). Rather than ideal objects existing *in* consciousness, we should say that consciousness exists *in* the world, insofar as it is conscious *of the world*, and *only* insofar as it is conscious of the world. This fundamental tenet of phenomenology is the reason for the equivalence of objects of consciousness and objects of the world—there are not, in fact, two objects, but one. (It is an object by definition, if it is something of which we are conscious—that does not mean it is “in” a subject, for that is what *makes it* an object.) Upon closer phenomenological scrutiny, some of Gödel’s most basic worries just disappear.

4. Concluding Remark

We went quite some distance. Actually, we have rushed through a difficult terrain with no time to stop and rest while still trying to point out some great vistas. The goal was to discover what Gödel might have found in Husserl with specific regard to mathematical realism. We have explored, in particular, the potential the language of phenomenology has for describing the experience that mathematicians deal with objects that are ideal, objective, and real. We saw, however, that Husserl is not a Platonist even though he may have sometimes used their language. This is what we see as one of

the main trajectories running through his life's work: learning how to naturalize Platonic hunches and how to translate their language into a language that can be accounted for in a sober scientific spirit. What Husserl therefore had to offer was not the rehabilitation of a naïve Platonism. Rather, what Husserl had to offer to Gödel was a reconciliation of subjective and objective idealisms. Husserl's contribution to establishing the objectivity of ideal objects is his insistence on the numerical identity between the object and the object of consciousness. Whether this amounts to a Platonism depends on what definition of "platonism" we adopt. Clearly, Husserl's reinterpretation is not a Platonic Platonism and a weak version only of Bernays' Platonism. For the criterion championed by Bernays, the "consciousness-independent existence," refers in Husserl either to validity, but not to basic concepts which owe their properties to the life-world, or to the independence a culture enjoys from its particular members, but then without any distinguished metaphysical aura.

Our final observation. In the beginning, we sketched the development within modern mathematics—we singled out the unrestricted use of the law of excluded middle and impredicative definitions—that made Bernays inject a new term to the philosophy of mathematics: Platonism. We know that Husserl himself was deeply influenced by these developments (see, e.g., Schmit 1981, Willard 1984, Lohmar 1989, or Centrone 2010 and the literature cited). Ironically enough, Husserl's general take on Platonism does not permit us to take a stance on the two issues that caused the whole debate. This would require a detailed phenomenological explication of what we find in certain mathematical acts. It was Gödel's hope that such a concentrated effort would lead to new axioms of set theory that would then settle open problems. He saw his hopes dashed, though.

NOTES

¹ "Euclide postule : on peut relier deux points par une droite ; tandis que M. Hilbert énonce l'axiome : deux points quelconques étant donnés, il existe une droite sur laquelle ils sont tous les deux situés. « Existe » vise ici le système des

droites. / Cet exemple montre déjà que la tendance dont nous parlons consiste à envisager les objets comme détachés de tout lien avec le sujet réfléchissant. / Cette tendance s'étant faite valoir surtout dans la philosophie de Platon, qu'il me soit permis de la qualifier du nom de « platonisme »." The symbol "*n*" indicates, here and in other quotes, a paragraph break in the original. Likewise, we use "*n* | *m*" to indicate a page break (from page *n* to page *m*) in the text we quote. We render emphasis in the original (spaced lettering, caps) as underlining.

² See d'Alembert (1765) and Brown (1991) for context; we owe Craig Fraser for the pointer to Brown.

³ See, e.g., Gray (2008, chs 2.1, 3.1, and 4.1) on geometry and Volkert (1986, esp. ch. I.6) on non-differentiable functions.

⁴ See, e.g., Corry (1996) on algebra and Ferreirós (1999) on set theory.

⁵ See Biermann (1969, *passim*); the German *Nostrifikation* meant (and in Austria still means) the approval of out-of-state transfer credits or diplomas.

⁶ Parsons (1995) gathers and discusses the evidence available on Gödel's realism; Tieszen (1992) collects what is known about Gödel and Husserl in general terms and sketches, in his (1998), Gödel's way to Husserl, a topic that was subsequently treated by van Atten & Kennedy (2003) in much more detail.

⁷ See, e.g., Gödel (2003d, 244): "The fact is that I have completed several versions [of the text], but none of them satisfies me [...] it may do more harm than good to publish half done work" (and so he did not). We owe this quote to Goldfarb (1995, 324).

⁸ See Gödel (1995c), (1995d), and (1995e) resp. Here and throughout we adopt the notation used in the edition of Gödel's collected works. Likewise, when there is no risk of confusion, we drop the name Gödel or Husserl from references.

⁹ See Bernays (1946) and Weyl (1946) for reviews of Gödel (1990b), and Kleene (1948), Jónsson (1948), and Buchdahl (1965) for reviews of Gödel (1990c) and Gödel (1990d), resp.

¹⁰ First in Wang (1974), later followed by Wang (1987), (1996); see Parsons (1998).

¹¹ See Wang (1987, 20, 112), and Wang (1974, 8–11), resp. These letters are now reproduced, in full, in Gödel (2003b, 396–399, 404f.). Russell's statement was known, however, to readers of his *Autobiography*.

¹² Gödel owned Husserl's *Logical Investigations* (1900-01), *Ideas*, vol. 1 (1913), *Cartesian Meditations* (1931), and *Crisis* (1936), copies he heavily annotated, but also shorter essays such as "*Philosophie als strenge Wissenschaft*" (1910) or the entry "Phenomenology" (1929) written for the *Encyclopedia Britannica*; see Føllesdal (1995, 367, note b), for details. He did not have a copy of *Formal and Transcendental Logic* (1929). Sometimes, we refer to these books by their acronyms LI, Id, FTL, CM, and C, and cite according to chapter and section. Usually, we offer both the German text and a translation, often our own. And since Husserl wrote in the Teutonic style of convoluted sentences, we often shrink an English quotation to the main assertion but indicate any omissions in the German original.

¹³ “[...] weil man nie den Mut hatte [...] der peinlichen Frage ins Angesicht zu sehen, wie die Subjektivität [...] Gebilde schaffen kann, die als ideale Objekte einer idealen ‘Welt’ gelten können.”

¹⁴ For our purposes we do not need to decide on the question whether or not there are conscious experiences that are non-intentional (e.g., moods or hollow urges). Husserl discusses the question in (Hua XIX, V, §15).

¹⁵ This is what links Husserl to Kant and makes him adopt the term “transcendental.” Kant argued that the objects of experience are not given in intuition but that their objecthood (e.g., unity) is mostly the result of a rational mind applying Aristotelian logic (in form of pure concepts) to what is given in intuition; objectivity then results from the validity of the logic involved. What Kant did for objects of experience, Husserl wants to do for abstract objects; including logic, which Kant took for granted.

¹⁶ Taxonomy is big, though; take botany, for example. (Thanks to Ben Datillo for educating us on this.) Naked-eye observation of plant morphology can be used for the purpose of taxonomy, but is not the most direct evidence, and can be misleading (similarities from convergence, differences from adaptations); a phylogenetic reconstruction based on the plant’s genetic make-up is more defensible. A rough, “naked-eye” taxonomy of conscious phenomena was a by-product of Husserl’s investigations. But when it comes to the classification of all conscious phenomena we miss a scientifically sound approach comparable to what phylogeny is for botany. It is not even clear what the most appropriate definition consciousness is. We think humility is the appropriate response, not to stop trying.

¹⁷ Husserl describes the natural attitude: “I am conscious of a world endlessly spread out in space, endlessly becoming and having endlessly become in time. I am conscious of it: that signifies, above all, that intuitively I find it immediately, that I experience it. By my seeing, touching, hearing, and so forth, and in the different modes of sensuous perception, corporeal physical things with some spatial distribution or other are *simply there for me, ‘on hand’* in the literal or the figurative sense, whether or not I am particularly heedful of them and busied with them in my considering, thinking, feeling, or willing.” (Husserl 1982, §27, 51).

¹⁸ See Schwitzgebel (2014) for an overview of the trouble with introspection.

¹⁹ « *Il est sensible, en effet, que, par une nécessité invincible, l’esprit humain peut observer directement tous les phénomènes, excepté les siens propres. Car, par qui serait faite l’observation? [...³⁰ | ³¹...] L’individu pensant ne saurait se par tager en deux, dont l’un raisonnerait, tandis que l’autre regarderait raisonner. L’organe observé et l’organe observateur étant, dans ce cas, identiques, comment t’observation pourrait-elle avoir lieu?* »

²⁰ For an overview, see Gallagher & Zahavi (2014).

²¹ See Henrich et al. (2010) for the notion of WEIRD people and Klein et al. (2018) for the latest batch of research efforts in this direction.

²² See, e.g., Kim (2011), ch. 2, for a discussion of such Cartesian privileges.

²³ See, e.g., the second volume of Husserl’s *Ideas*, composed 1912–28 but published in 1952

²⁴ See Hua XIX, V, Appendix to § 11 and § 20, 439.12: “Man braucht es nur auszusprechen [...] daß der intentionale Gegenstand der Vorstellung derselbe

ist wie ihr wirklicher und gegebenenfalls ihr äußerer Gegenstand und daß es widersinnig ist, zwischen beiden zu unterscheiden.” Most emphasis suppressed in translation; translation by Findlay.

²⁵ “Such errors have dragged on through the centuries – one has only to think of Anselm’s ontological argument – they have their source in factual difficulties, but their support lies in equivocal talk concerning ‘immanence’ and the like.” (Husserl 1970, 127 ; Hua XIX, V, Appendix to 11 and 20, 595)

²⁶ The two main sources from his published writings is CM, Med V, and his essay “On the origin of geometry,” published as an appendix to *Crisis*. His manuscripts on intersubjectivity 1905–1935 were collected and published as *Husserliana*, vol. XIII–XV.

²⁷ To be clear, it is not the failure to prove that the world and other people in it exist; this much is taken for granted. Rather, the menace is that phenomenology might turn out not to be the all-encompassing first philosophy it claims to be in case its methods should fail to account for the constitution of an objective world and people with a shared intersubjectivity in it.

²⁸ “In der Abstraktion verbleibt uns eine einheitlich zusammenhängende Schicht des Phänomens Welt.”

²⁹ “So gehört zu meiner Eigenheit als von allem Sinn fremder Subjektivität gereinigte, ein Sinn bloße Natur [... unter deren Körpern] finde ich dann in einziger Auszeichnung meinen Leib, nämlich als den einzigen, der nicht bloßer Körper ist, sondern eben Leib, das einzige Objekt innerhalb meiner abstraktiven Weltschichte, dem ich erfahrungsgemäß Empfindungsfelder zurechne.”

³⁰ Hua I, §49, 137.1ff.: “Der Seinssinn objektive Welt konstituiert sich auf dem Untergrunde meiner primordinalen Welt in mehreren Stufen. Als erste ist abzuheben die Konstitutionsstufe des Anderen [...] Damit in eins und zwar dadurch motiviert vollzieht sich eine allgemeine Sinnesaufstufung auf meiner primordinalen Welt, wodurch sie zur Erscheinung von einer bestimmten objektiven Welt wird [...] und] letztlich eine Monadengemeinschaft [...] / deren] transzendente Intersubjektivität [...] die objektive Welt intersubjektiv konstituiert.”

³¹ Since appresentation is a special kind of apperception, we could keep terminology simple and use apperception throughout. We use, however, appresentation and its derivatives wherever we feel it is helpful to recall the act of co-presenting a consciousness with another human body.

³² “Zum Seinssinn der Welt und insbesondere der Natur als objektiver gehört ja [...] das *Für-jedermann-da*, als von uns stets mitgemeint.”

³³ What we called “full intersubjectivity” is Husserl’s notion of a “transcendental intersubjectivity:” an open community of exchange among peer human consciousnesses, or monads; see *Cartesian Meditations* (Hua I §49, esp. pp. 137–138, and §§56–58, *passim*).

³⁴ Hua I, §43, 124.11ff.: “Zudem gehören zur Erfahrungswelt Objekte mit geistigen Prädikaten, die [...] auf fremde Subjekte [...] verweisen: so alle Kulturobjekte (Bücher, Werkzeuge und Werke irgendwelcher Art usw.), die dabei aber zugleich den Erfahrungssinn des Für-Jedermann-da mit sich führen (scilicet für Jedermann der entsprechenden Kulturgemeinschaft, wie

der europäischen, eventuell enger: der französischen etc.).” See also his remarks on cultural predicates (*Kulturprädikate*) and alien spirituality (*Fremdgeistiges*), *ibid.* §44, 126f.

³⁵ Husserl calls it a *Sprachleib* and *Verleiblichung* (*ibid.*, 369.5f.), which is a powerful metaphor in German. For the German word *Leib*—unlike *Körper*, which is the translation of (Platonic) “solid” or “field” (in algebra)—can only denote the body of a higher organism. Husserl thus suggests that the linguistic body of a fleeting product of consciousness is somehow comparable to the human body that grants consciousness persistence through time. Derrida (1962, 69) emphasizes the same point when he renders *Sprachleib* as *la chair linguistique* (linguistic flesh).

³⁶ Husserl is clearly biased towards written records; see (Husserl 1976a, 371.26). He was inattentive to the feats of memorization that are common in oral cultures; see, e.g., Kelly (2015).

³⁷ “Wie steht es mit der Objektivität dieser idealen Gebilde, dieses Apriori [der Geschichte], wie mit seiner Objektivität? Da kommen wir wieder auf die Voraussetzung [... einer] nicht abbrechenden Tradition.”

³⁸ “[D]ie gesamte Kulturgegenwart [...] ‘impliziert’ die gesamte Kulturvergangenheit [...] Genauer gesprochen, sie impliziert eine Kontinuität einander implizierender Vergangenheiten [.../....] das universale Apriori der Geschichte.”

³⁹ This is how we understand *ibid.*, 366.20: “In einer Unzahl von Traditionen bewegt sich unser menschliches Dasein. Die gesamte Kulturwelt ist nach allen ihren Gestalten aus Tradition da.”

⁴⁰ “Nur durch Rekurs auf dieses [...] zu entfaltende Apriori können unsere apriorischen Wissenschaften [...] eine ernstlich wissenschaftliche Begründung gewinnen.”

⁴¹ “[D]ie vermeintlich völlig eigenständige Logik [...] als universale apriorische Fundamentalmissenschaft [...] ist nicht anderes als eine Naivität. Ihre Evidenz entbehrt der wissenschaftlichen Begründung aus dem universalen lebensweltlichen Apriori.”

⁴² Plato’s exposition of mathematical objects in the hierarchy of objects appears at 510c–511b in the *Republic*. At 511a, Socrates says (*Works*, tr. Paul Shorey): “This then is the class that I described as intelligible, it is true, but with the reservation first that the soul is compelled to employ assumptions in the investigation of it, not proceeding to a first principle because of its inability to extricate itself from and rise above its assumptions, and second, that it uses as images or likenesses the very objects that are themselves copied and adumbrated by the class below them, and that in comparison with these latter are esteemed as clear and held in honor.” The Greek reads, Plato, *Opera*:

“τοῦτο τοίνυν νοητὸν μὲν τὸ εἶδος ἔλεγον, ὑποθέσει δ’ ἀναγκαζομένην ψυχὴν χρῆσθαι περὶ τὴν ζήτησιν αὐτοῦ, οὐκ ἐπ’ ἀρχὴν ἰούσαν, ὡς οὐ δυναμένη τῶν ὑποθέσεων ἀνωτέρω ἐκβαίνειν, εἰκόσι δὲ χρωμένην αὐτοῖς τοῖς ὑπὸ τῶν κάτω ἀπεικασθεῖσιν καὶ ἐκείνοις πρὸς ἐκεῖνα ὡς ἐναργεῖς δεδοξασμένοι εἰτε καὶ τετιμημένοι.”

⁴³ “Die Geltung dieser Gesetze [...] hängt nicht davon ab, ob wir und wer immer begriffliche Vorstellungen faktisch zu vollziehen [...] vermag.” *Ibid.*

§36, 125.9ff.: “Was wahr ist, ist absolut, ist “an sich” wahr; die Wahrheit ist identisch eine, ob sie Menschen oder Unmenschen, Engel oder Götter urteilend erfassen.” Ibid. §39, 136.6–9: “Aber jede Wahrheit an sich bleibt, was sie ist, sie behält ihr ideales Sein [...] Sie gehört zum Bereich des absolut Geltenden.”

⁴⁴ “Beiderseits handelt es sich um Begriffe, die, [...] nur im Hinblick auf die verschiedenen ‘Denkfunktionen’ ²⁴⁵|²⁴⁶ entspringen, d.h. in möglichen Denkakten [...] ihre konkrete Grundlage haben können.”

⁴⁵ “Gesetze [...] die unmittelbar in den kategorialen Begriffen wurzeln.”

⁴⁶ “[A]ndererseits ist es von vornherein klar, daß ihre Deduktion [...] ausschließlich in jenen Theorien fassen muß.”

⁴⁷ “[W]ie das Sein oder Gelten von Allgemeinheiten auch sonst den Wert von idealen Möglichkeiten besitzt [... so] auch hier: Die Aussagen ‘die Wahrheit gilt’ und “es sind denkende Wesen möglich, welche Urteile des bezüglichen Bedeutungsgehaltes einsehen’, sind von gleichem Werte.”

⁴⁸ Here we wish to point out that additions he made post-1960 to an earlier paper seem directly informed by Husserl’s philosophy; see Gödel (1990d, 268).

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