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Weyl's Phenomenological Constructivism

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Abstract

Several scholars support the view that Weyl's investigations have undergone many changes along his life. Among them, there is no common agreement, but many authors set an early phase connected with Weyl's adherence to intuitionism and a later phase usually referred as Weyl's symbolic constructivism. The paper aims to show that previous interpretations are reasonable though they miss the phenomenological framework in which they can be better understood. I will focus on some Husserlian issues that I think were overlooked in the literature. Husserl's distinction between descriptive and exact concepts delineates the difference between a descriptive analysis of a field of inquiry and its exact determination. Clarifying how they are related is not easy. Nonetheless, the proposed difference between descriptive and exact sciences does not exclude the fact that they might coexist as two correlated investigations in the same field of inquiry once we were able to establish a connection by means of some idealizing procedure intuitively ascertained. A uniform interpretation of Weyl's investigations is proposed within Husserl's phenomenological framework, at least in the period 1917-1927.

Keywords: Weyl, Husserl, phenomenology, constructivism, descriptive, exact, limiting ideas, space, continuum

Hermann Weyl (1881-1955) was a leading mathematician at the beginning of the twentieth century. His major contributions concerned several fields of research, both in pure mathematics and theoretical physics, and, most importantly, his pioneering work was carried out in the light of his unique philosophical view. As only few mathematicians of his time, Weyl dealt with both scientific and philosophical issues with great skill, becoming a very unique figure among scientists and mathematicians of his time.[1] Although Weyl was a well-known mathematician, philosophers of mathematics have started getting interested in his work only recently, and, even though several authors have tried to uncover the philosophical framework that underlies Weyl's studies, many of them did not identify a coherent perspective in his philosophical view, arguing that his foundational research changed over the years.

Both Sieroka (2009) and Mancosu (2010) recognize at least two main tendencies in Weyl's work between 1917 and 1927: A first phase mainly characterized by his criticisms against set theory and classical analysis, and by his rejection of Hilbert's formalism and adherence to the intuitionistic-oriented account of Husserl and then Brouwer; and a second phase characterized by his tendency toward a sort of symbolic constructivism and his reconciliation with Hilbert's formalism. A similar interpretation is also supported by Da Silva (1997), Bell (2004), and Folina (2008), who identify a changeable perspective over the period 1917-1927, which went from an intuitionistic-oriented approach to a constructivist account.[2]

Not everyone agrees with the interpretation that supports a perspective changing over time. For instance, Scholz gives a more uniform interpretation of Weyl's research, and, in Scholz (2000), he suggests a constructive reading of Weyl's work since the publication of *Das Kontinuum*, arguing that Weyl was strongly influenced by Fichte's constructive philosophy. A similar constructive interpretation is also defended by Tieszen (2000), although he takes into account also the influence of Husserl's philosophy by suggesting that the philosophical framework of Weyl's mathematical constructivism should be understood in the light of transcendental idealism, which finds its roots in Kant, Fichte, and Husserl. Thus, Tieszen (2000) proposes a strong constructivist reading of Husserl's philosophy.

This paper aims to support a more uniform interpretation of Weyl's research in the period 1917-1927.[3] We will focus on three main works: *Das Kontinuum* (1918), *Raum-Zeit-Materie* (1921), and *Philosophy of Mathematics and Natural Science* (1949).[4] In certain respects, my interpretation will be close to Tieszen's reading, although it will also highlight some important Husserlian issues that in my opinion were previously overlooked.

1. The Mathematical Form of the Euclidean Space

Weyl's research on the nature of intuitive space constitutes an important body of work. The space of intuition pertains to our experience of spatiality and it should not be confused with any of its conceptualization. We "have to differentiate carefully between phenomenal knowledge or insight", and "theoretical construction" (Weyl 1949, 61): The first is conveyed by statements like "this leaf (given to me in a present act of perception) has this green color (given to me in a present act of perception)" (Weyl 1949, 61); on the other hand, the second is characterized by rational principles and it allows us to "'jump over its own shadow', to leave behind the stuff of the given, to represent the *transcendent*" (Weyl 1949, 66). Mathematics and physics allow us to achieve this sort of theoretical construction, and Weyl's mathematical formulation of *affine geometry* is an attempt in this direction. Indeed, he aims to develop a mathematical account of our intuitive space that is not "demanding the reduction of all truth to the intuitively given" (Weyl 1949, 65).

For any intuitively given field of inquiry, we should be able to first identify the *basic categories of objects* (*Grundkategorien*) and the *primitive relations* among these objects (*ursprünglichen Relationen*) that pertain to it.[5] A *primitive judgment scheme* (*ursprüngliche Urteilsschema*) is associated with each primitive relation, which "yields a meaningful proposition" only when each blank of the relation is filled by an object of its corresponding category (Weyl 1994, 41). In the first part of *Das Kontinuum* Weyl, deals with this subject matter and gives some examples. The proposition "this leaf is green", whose judgment scheme is "G(x): x is green", is meaningful (*sinnvoll*) because the blank x is affiliated with the category "visible thing" and it is filled by the object "leaf", which is indeed a visible thing (Weyl 1994, 5).[6] Weyl aims to avoid any mathematical account that makes use of judgment schemes that yield meaningless propositions. He remarks that "anyone who forgets that a proposition with such a structure can be meaningless is in danger of becoming trapped in absurdity" (Weyl 1994, 6).[7] For this reason, Weyl takes into consideration only "well-structured" primitive judgment schemes, from which further judgment schemes can be derived by applying certain principles of logical construction, without bringing again intuition into play. Weyl refers to these judgement schemes as *complex judgment schemes* and calls *derived relations* the associated relations. What sort of new judgment schemes "will unfold before our intuition in the development of the life of the mind can certainly not be anticipated *a priori*" (Weyl 1994, 113).[8] Despite this, the principles of logical construction "can be set down once and for all (just like the elementary forms of logical inference)" (Weyl 1994, 113).[9] Among these principles, Weyl identifies the judgments that express a state of affairs regarding the given field of inquiry: They are called *pertinent judgements* and they allow us to acquire a "complete knowledge of the objects of the basic categories as far as they are connected by the basic relations" (Weyl 1949, 7). Therefore, a meaningful mathematical analysis of an intuitively given field of inquiry starts with the identification of its basic categories and primitive relations. A mathematical theory can then be logically built on them, without bringing again intuition into play.

On this basis, Weyl develops the affine geometry and identifies two "fundamental categories of objects", namely *spatial-point category* and *translation category* (Weyl 1952, 18). Weyl also refers to them as the category of *points* and the category of *vectors*, respectively. Few primitive relations are found among these objects, i.e. the *axioms* concerning the operations of *addition* and *multiplication*, and the relationships between points and vectors. Weyl then points out that all concepts that may be defined, only by using logical reasoning from the basic notions of vector and point and their primitive relations "belong to affine geometry" (Weyl 1952, 18). For instance, it is possible to define the concept of a *straight line* and a *plane*:

- given a point O and a vector $\vec{e}$, the end-points of all vectors $\overrightarrow{OP}$ which have the form $\lambda\vec{e}$ constitute a *straight line*;

- given a point O, a vector $\overrightarrow{e_1}$, and a vector $\overrightarrow{e_2}$ which is not of the form $\lambda\overrightarrow{e_1}$, then the end-points of all vectors $\overrightarrow{OP}$ that have the form $\lambda_1\overrightarrow{e_1} + \lambda_2\overrightarrow{e_2}$ constitute a *plane*.

It is then possible to derive the totality of all possible formations concerning that field of inquiry from a few basic notions and relations. Moreover, all theorems that can be logically deduced within this framework constitute "the doctrine of affine geometry" (*Lehrgebäude der affinen Geometrie*) (Weyl 1952, 18). In this sense, geometry turns out to be a "*theory of space*" (Weyl 1949, 18).[10]

Furthermore, Weyl introduces the notion of *n-dimensional linear vector-manifold* (*n-dimensionale lineare Vektor-Mannigfaltigkeit*), which consists of all vectors of the form $\lambda_1\overrightarrow{e_1} + \cdots + \lambda_n\overrightarrow{e_n}$ (where $\overrightarrow{e_1}, \dots, \overrightarrow{e_n}$ are n linearly independent vectors, i.e. their linear combination only vanishes when all the coefficients vanish).[11] Affine geometry is obtained when n=3. He then formulates the last axiom of affine geometry, the *dimensional axiom*, which states that in affine geometry (3-dimensional linear manifold) there are three linearly independent vectors, but every 4 vectors, the vectors become linearly dependent on one another.[12]

However, this mathematical conceptualization is not unique: Any field of inquiry allows us to identify only certain categories of objects or primitive relations but their choice can be "arbitrary to a considerable extent" since they are not uniquely determined by the field of inquiry (Weyl 1949, 20). The difference between "essentially originary and essentially derived notions lies beyond the competence of the mathematician" (Weyl 1949, 20). The classical concept of space that concerns *Euclidean geometry* provides another possible conceptualization of the space of intuition. Specifically, Euclidean geometry is able to account for its homogeneity. In this case, we deal with three categories of objects, *spatial-point*, *line*, and *plane*, that are not defined but rather "assumed to be intuitively given" (Weyl 1949, 3). Few primitive relations are associated with these categories: *incidence*, *betweenness*, and *congruence*. Weyl also remarks that the category of points "reflects the intuitive homogeneity of space" (Weyl 1949, 8).

Indeed, any judgment scheme "P(x)" with blank x relating to this category and derived from the primitive judgement schemes without any reference to individual spatial-points, lines or plane "is always true either of *each* or of *no"* point (Weyl 1994, 16). For instance, the property "P(x): there exists a line such that the point x lies on it" is always true for any given point. On the other hand, the property "P(x): there exist three points y_1, y_2, y_3 lying on a line (y_2 being between y_1 and y_3) such that x is between y_1 and y_2 and it is also between y_2 and y_3" is always false. For this reason, Weyl refers to this category as a *homogeneous category*. Therefore, this mathematical conceptualization allows us to account for the intuitive homogeneity of space.

Although Weyl acknowledges the possibility of different conceptualizations of the space of intuition, the choice of which conceptualization to opt for is somehow limited; indeed, in some cases we should prefer one conceptualization over another. For instance, the axiomatic construction of affine geometry seems to be a better conceptualization of the space of intuition as it consists of "a system that, also in logical respect, is of a much more transparent and homogeneous structure than the purely geometrical axioms of Euclid or Hilbert" (Weyl 1949, 69). This theoretical construction reveals "a wonderful harmony between the given on one hand and reason on the other" (Weyl 1949, 69). Moreover, the derived concepts of straight line and plane "correspond to those which suggest themselves most naturally from the logical standpoint" (Weyl 1949, 69). For these reasons, Weyl claims that affine geometry best conceptualizes what is intuitively given.

To conclude, we will sum up the main features that characterize Weyl's studies. His research implies a distinction between two kinds of knowledge: The first concerns our sense perception, and Weyl refers to it as a phenomenal knowledge; the second seems to pertain to a domain of mathematical concepts, and Weyl refers to it as a sort of theoretical construction. Although being two different kinds of knowledge, Weyl seems to believe in the possibility of establishing a connection between them. He attempts to formulate a mathematical conceptualization of the space of intuition starting from few basic notions and relations that are intuitively grasped. However, our mathematical knowledge of the real world is not limited to this intuitive source of knowledge, but it is logically built on the basic notions and relations without bringing again intuition into play. This is how the mathematical knowledge of real world can represent the transcendent. Finally, Weyl suggests that different mathematical conceptualizations are possible, but deciding which approach to adopt is not a matter of choice, and one conceptualization might be preferred to another. This arises the problem of finding which mathematical conceptualization best suits what is intuitively given.

2. The Continuum

In *Raum-Zeit-Materie* Weyl remarks that his axiomatic formulation of affine geometry is still far from being satisfactory since it lacks a proper understanding of continuity. In *Das Kontinuum* Weyl does not deduce the notion of multiplication and the related laws from the principles of addition because the axioms of multiplication "cannot be derived in the general form from the axioms of addition by logical reasoning alone" (Weyl 1952, 17). The continuum "is so difficult to fix precisely, from the logical structure of geometry" (Weyl 1952, 17).[13] For this reason, Weyl deals with the nature of continuum in several works aiming to better understand the issue. In *Das Kontinuum*, for instance, Weyl explores the extent to which our mathematical theories of space and time reflect the intuitive content that we experience. Since we experience them as two continuous entities, our mathematical theories should reflect their continuous nature. Hence, understanding the nature of continuum turns out to be especially important for understanding the real world. It contributes "to critical epistemology's investigation into the relations between what is immediately (intuitively) given and the formal (mathematical) concepts through which we seek to construct the given in geometry and physics" (Weyl 1994, 2).

We shall now focus on the mathematical formulation of these continua as it is developed in *Das Kontinuum*. Weyl clarifies that the object of his investigation is the *phenomenal continuum*, be it spatial or temporal. By temporal continuum he means the constant form of our experiences of consciousness by virtue of which they appear to us to flow by successively. He further explains that by experience he does not mean "real psychical or even physical processes" which occur in an individual, "belong to a real world and, perhaps, correspond to the direct experiences". He means what we experience, exactly how we experience it (Weyl 1994, 88). Thus, the phenomenal time should be understood as a pure experience, it refers to the direct perception that we have of it, and it should not be confused with the time of physics or with any other notion of time derived from a certain view of the world.[14] Weyl aims to develop a mathematical theory of the phenomenal continuum. In order to do this, we need first to identify which kinds of basic categories and primitive relations belong to this field of inquiry. However, this is not easy and Weyl needs to postulate the possibility that a "now" is intuitively given in order to have "some hope of connecting phenomenal time with the world of mathematical concepts" (Weyl 1994, 88). By making this assumption, we are able to dissolve the phenomenal time into isolated time-points, rigidly punctual "now", and then, by identifying this sequence of time-points, we can grasp this species of time in an exact way. The time-points belong to a basic category – the *time-point category* – and the following primitive relations can be associated with them:

- the binary relation E_{arlier}(A,B): A is earlier than B;
- the quaternary relation E_{qual}(A,B,A',B'): A is earlier than B, A' is earlier than B', and AB is equal to A'B'.[15]

A mathematical theory of time could be logically built on the above-mentioned basic category and primitive relations, but first some issues must be solved. Indeed, these relations are not sufficient to conceptually differentiate every time-point in the given continuum. The phenomenal time is homogeneous and, as in the case of the homogeneity of the space of intuition, it can be shown that any judgment scheme (whose blank x is associated with the time-point category) that is derived from the primitive judgement schemes without any reference to individual time-points is always true either of each point or for none. Therefore, a single time-point "can only be given by being specified individually", i.e. by a direct intuition (Weyl 1952, 8). There is no intrinsic property that we can assign to a specific time-point in order to differentiate it from all the others.

According to Weyl, the issue could be solved by establishing an *isomorphism* between the domain of time-points and the domain of real numbers (as they are constructed in *Das Kontinuum*).[16] Each time-point will then be associated with a definite real number and vice versa. Specifically, we first need to fix two time-points, O and E, by means of a direct intuition such that $E_{arlier}(O,E)$ holds true. Then we can "fix conceptually further time-points P by referring them to the unit-distance OE" (i.e. the time span OE taken as unit) (Weyl 1952, 8). This is done by establishing a connection between a time-point P and the relation $R_t(P,O,E)$ that can be expressed in the form $OP=t*OE$. Our mathematical theory of time will have the same structure of real numbers, if this relation, logically derived from the primitive relations, reflects Weyl's construction of a real number. In this case, we could establish an isomorphism between the two domains, and we could associate a real number t with each time-point P. Moreover, Weyl speaks of *co-ordinate system* centered at O (with OE being the unit of length), where t represents the *abscissa* with respect to this co-ordinate system.[17] Weyl further points out that this conceptualization of phenomenal time relies on the individual exhibition of the time-point O. Only through this intuitive act we are able to differentiate time-points in the temporal continuum. Weyl claims that this fact is due to "the unavoidable residue of the eradication of the ego" in that theoretical construction of the world whose existence can only be given "as the intentional content of the processes of consciousness of a pure, sense-giving ego" (Weyl 1994, 94).[18]

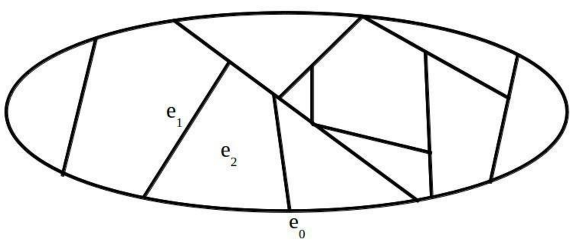
If we can indeed establish an isomorphism, we should be able to confirm it by direct inspection of phenomenal time. That is to say, our intuition should confirm "whether this correspondence between time-points and real numbers holds or not" (Weyl 1994, 90). However, our "intuition of time provides no answer" (Weyl 1994, 90). We face this situation because such interrogation is meaningless: Our mathematical theory of time, indeed, fails to satisfy a fundamental criterion of any theoretical construction, i.e. the time-point category "lacks the required support in intuition", no judgment scheme involving this category can be filled by time-points given by an individual intuition (Weyl 1994, 90). What is given in consciousness presents itself "not simply as a being" but "as an enduring and changing being-now" (Weyl 1994, 91). This being-now is "in its essence, something which, with its temporal position, slips away" (Weyl 1994, 92). For this reason, a mathematical theory of time that dissolves the phenomenal time into time-points turns out to be inadequate. This is due to the continuous nature of phenomenal time: A time-point "exists only as a 'point of transition' […] always only an *approximate*, never an *exact* determination is possible" (Weyl 1994, 92).[19]

Similar observations are also put forward in regard to the spatial continuum. In *Das Kontinuum*, Weyl deals with the phenomenal continuum of spatial extension and, by following his previous work *Die Idee der Riemannschen Fläche*, he attempts to conceptualize the continuous connectedness of the points on a two-dimensional surface. Since he needed to postulate that a "now" is intuitively given, he now needs to assume that an exact "here" can be fixed. However, the continuum does not consist of isolated individual points, and a fixed spatial-point "cannot be exhibited in any way", meaning that an exact determination is never possible (Weyl 1994, 92). Moreover, Weyl acknowledges that additional problems arise even if we accept this postulate. Indeed, we can regard a spatial surface as a "two-dimensional manifold" of surface-points, whose continuous connectedness can be grasped by means of the notion of *neighborhood* (Weyl 2009, 16). Given two surface-points P and Q, and a relation N that satisfies certain conditions, we say that Q lies in the n-neighborhood of P, if the relation N(P,Q;n) holds. This relation aims to represent the structural properties involved in the common notion of neighbourhood $|x - x_0| < n$, so that all ideas of continuity in a two-dimensional surface can be developed within this abstract scheme, free from intuitive knowledge. Although this approach offers many advantages, reducing the continuous connectedness to the concept of neighbourhood is not satisfactory. When a relation N(P;Q;n) establishes the n-neighbourhood of P, "much more occurs than is given by the continuous connectedness itself" (Weyl 1994, 106). In the case of the plane, for instance, "we could choose the interior of the circle of radius 1/n about a point as the nth neighbourhood of that point, but the circle of radius $1/2^n$ would serve just as well" (Weyl 1994, 107). We could also employ several other shapes in place of the circular ones (elliptical, square, etc.). No clear-cut answer "is yet at hand to the question of how we shall establish the link between the given and the mathematical in a perspicuous manner" (Weyl 1994, 107). Thus, dissolving the phenomenal continuum into isolated spatial-points turns out to be deeply unsatisfactory.[20]

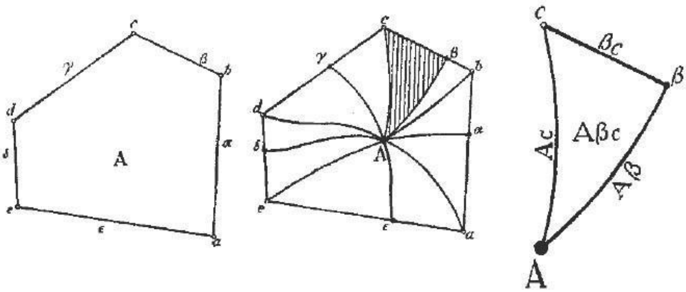
Weyl's studies on the nature of space and time are not pointless, on the contrary, they are of great importance for our understanding of the real world. The abstract schemata of our mathematical theories "must underlie the exact science of domains of objects in which continua play a role" (Weyl 1994, 108). Weyl indeed believes that a sort of "Logos" dwells within reality and we can try to reveal it as much as possible. Our mathematical theories are not a matter of choice just like "our inability to connect up the continuous with the schema of the whole numbers is not just a matter of personal preference" (Weyl 1994, 93, note 11). In this sense he claims that his construction of analysis "contains a *theory of the continuum* which must establish its own reasonableness (beyond its mere logical consistency) in the same way as a physical theory" (Weyl 1994, 93).

After the publication of *Das Kontinuum*, Weyl revises his mathematical approach to the continuum. His first approaches rely on the assumption that it is possible to exhibit a time-point or spatial-point in an individual intuition. However, this assumption violates the essence of continuum, which, by its very nature, cannot be shattered into a multitude of individual elements. The relation between parts and the whole, and not the relation between each element and the set of elements, should underlie the analysis of the continuum.[21] The continuum "falls under the notion of the 'extensive whole', which Husserl describes as what 'permits a dismemberment of such a kind that the pieces are by their nature of the same lowest species as is determined by the undivided whole'" (Weyl 1949, 52).[22] Weyl first tries to improve his approach to the continuum in *Über die neue Grundlagenkrise der Mathematik* published in 1921 (transl. Weyl 1998). In this paper, Weyl emphasizes the "the inner groundlessness of the foundations" upon which the current mathematics rests (Weyl 1998, 86). By following Brouwer's ideas, he then attempts a different approach to the concept of real number and continuum. At that time, Weyl was deeply impressed by the work of Brouwer and his foundational viewpoint: he states: "[…] Brouwer – that is the revolution!" (Weyl 1998, 99).[23] In the last pages of the *Über die neue Grundlagenkrise der Mathematik*, Weyl stresses the need for a different mathematical approach to the continuum of a two-dimensional manifold.

He first formulates the schema S concerning the *topological structure* of the manifold. It consists of finitely many *corners* e_0 (elements of level 0), *edges* e_1 (elements of level 1) and *surface pieces* e_2 (elements of level 2).



Few basic properties can be established: Each surface is limited by certain edges and each edge by certain corners. These properties represent the content of the schema S, i.e. the *topological framework* of the manifold. This schema "has to satisfy certain requirements, which can easily be stated" (Weyl 1998, 115). Weyl then outlines a *process of division* by dividing each edge into two edges by means of one of their points. Analogously, each surface piece is divided into triangles by using lines that start from a center, arbitrarily chosen within the surface piece, and that are connected to the corners of the surface piece.[24]



The figure is an example of how a surface piece, in this case a pentagon, is divided, and it shows the first step of the process of division from S to S'. We can easily identify the elements resulting from the process of division. For instance, the edge β is divided by setting an arbitrary point that leads to the generation of two new edges, namely βc and βb. In addition, an arbitrary point set within the surface piece A is used to divide the surface piece into triangles, obtaining the new surface piece Aβc. All other elements can be identified in a similar way and then properly named. Weyl then points out that we can identify a general pattern: Given the initial schema S, any symbol $e_2e_1e_0$ represents a surface piece e'_2 of the subdivided scheme S': through the iteration of this symbolic process, we obtain a sequence of derived schemes S,S',S'',S''', etc. so that what "we have done is nothing else than devise a systematic cataloguing of the parts created by consecutive subdivisions" (Weyl 2012, 76). The *sequence* ee'e'' and so forth pinpoints a point in the continuum; the sequence starts with a surface piece e of S and provides that the surface piece $e^{(n)}$ of the scheme $S^{(n)}$ is followed by a surface piece $e^{(n+1)}$ of $S^{(n+1)}$, leading to the further division of $e^{(n)}$. From the surface pieces of the initial topological framework, i.e. the schema S, we then reach the points of the manifold by iterating the process of division infinitely many times. This mathematical conceptualization is able to account for the essential feature of the continuum, which relies on the relation between part and whole, where "*every part of it can be further divided without limitation*" (Weyl 1998, 115). A point in a manifold must be seen as a *limiting idea* (*Grenzidee*): the concept of a point is indeed "the idea of the *limit* of a division extending *in infinitum*" (Weyl 1998, 115). Hence, Weyl believes that everyone feels "how truly the new analysis conforms to the intuitive character of the continuum" (Weyl 1998, 117).[25]

To conclude, Weyl's most recent studies on the nature of the continuum seem to be similar to his previous investigations. Indeed, they both underlie a distinction between two kinds of knowledge, one related to sense perception and one concerning the domain of mathematical concepts. Also, what is intuitively given seems to be the starting point of both approaches. Our mathematical understanding of the continuum should rely on intuitive insight, and a theoretical construction should be developed on the basis of basic notions and relations that are intuitively given. Moreover, our mathematical conceptualizations are not a matter of choice: Weyl seems to suggest that a sort of "Logos" dwells into reality and that these studies allow us to grasp the abstract schemata that underlie what is immediately given. However, the analysis of the continuum turns out to be more complicated as the assumption that it is possible to exhibit a time-point or a spatial-point in an individual intuition arises several problems. Hence, Weyl aims to improve his analysis of the continuum in later research, and, specifically his work in topology addresses the issue by regarding a point in a continuum as a limiting idea, i.e. the idea of the limit of a division extending *in infinitum*. According to Weyl, this approach is a more faithful analysis of the continuum. Thus, Weyl's research seems to be characterized by a constant search for the mathematical conceptualization that best suits what is intuitively given.

3. A Phenomenological Framework

Weyl's studies can be better understood within the philosophical framework of Husserl's phenomenology. Edmund Husserl (1859-1938) came to Göttingen as *extraordinarius* professor of philosophy in 1901. In 1904, Weyl moved to Göttingen to study mathematics and physics, and he received his doctorate in 1908 under Hilbert's supervision. Therefore, in the years 1904-1913 Husserl and Weyl worked at the same university. Historical records show that they knew each other, but Weyl's interest in phenomenology was actually sparked by his future wife Helene Joseph (1893-1948). Helen moved to Göttingen to become a student of Husserl in 1911, and, since then, her philosophical thinking was deeply influenced by phenomenology. In the years following the period in Göttingen, Weyl and his wife became friends with Husserl and his family and, when Husserl's youngest son fled Germany during Nazism, he was hosted for some time by the Weyls in Princeton.[26] Weyl sent a copy of *Das Kontinuum* and *Raum-Zeit-Materie* to Husserl, who in turn sent a copy of the second edition (revision of the sixth logical investigation) of *Logische Untersuchungen*. Four letters from Husserl to Weyl have been preserved, and they clearly provide evidence of the Weyl's close affiliation with phenomenology during the years 1917-1927.[27]

Weyl's studies that we have shown strongly suggest a Husserlian influence, since several Husserlian issues underlie Weyl's investigations, such as Weyl's distinction between phenomenal and conceptual knowledge and his theory of meaning. Moreover, Weyl makes explicit reference to Husserl's writing several times. As mentioned in the introduction, several works in recent literature have shown Husserl's influence on Weyl's scientific investigations. I will now focus on some Husserlian issues that I think were overlooked in the literature, and I will put forward a more uniform interpretation of Weyl's studies in the period 1917-1927.

In *Ideen I*, Husserl emphasizes a distinction between *descriptive sciences* and *exact sciences* and argues that, although they are both eidetic sciences, they are essentially different. Geometry is a good example of exact science as it is an *axiomatic science* that operates with *exact concepts*, which express *ideal essences*. Geometry derives every ideally possible spatial form starting with few basic concepts and by using few primitive axioms. However, all these "derived essences" are not usually intuited, which means that geometry does not grasp each essence directly but it derives them by mediate reasoning. For this reason, Husserl refers to exact science also as *explicative sciences*. Moreover, geometry "can be completely certain of dominating actually by its method all the possibilities and of determining them exactly" (Husserl 1982, 163). Husserl refers to this "fundamental logical property" in terms of *definite manifold* (Husserl 1982, 163).[28] A field of inquiry is articulated as a definite manifold if it is possible to derive all the possible formations concerning that field by starting from a few basic concepts and a given set of axioms. On the other hand, a descriptive science is purely descriptive and it operates with *inexact concepts*, which express *morphological essences*. A descriptive science investigates its field of inquiry though a direct seeing of essences. In this sense, we can refer to phenomenology as a descriptive science as its phenomenological descriptions are based on a direct seeing of essences.[29] Nonetheless, the proposed difference between descriptive and exact sciences does not exclude the fact that they might coexist as two correlated investigations in the same field of inquiry. A field of inquiry, for instance, might be articulated as a definite manifold. However, this fact is not a matter of choice and it "must be demonstrable in immediate intuition" (Husserl 1982, 165). One of the necessary conditions, for instance, has to be "the exactness in '*concept-formation*', which is by no means a matter of free choice and logical technique" (Husserl 1982, 165). The exactness of the basic concepts has to be grounded on the descriptive analysis of the field of inquiry itself, so that *their meaning is completely clarified* within this phenomenal domain. There must be some idealizing procedure, intuitively ascertained, that replace morphological essences with ideal essences. Husserl further points out that these ideal essences, grasped by such an idealization, have to be considered as a sort of "limit", that is *limiting ideas* (*Grenzideen*) in the Kantian sense. In this way, it might be possible to regard a field of inquiry as a definite manifold.[30] An important case is represented by the relationship between intuitive space and geometry. The former is extensively described by Husserl's eidetic investigations on our spatial experience: these phenomenological descriptions constitutes a *descriptive material eidetic science of space*. On the other hand, the latter is an eidetic science dealing with all possible *spatial forms* by means of exact concepts, that is an *exact material eidetic science of space*. Clarifying all connections between these two sciences is not an easy task. In *Ideen I*, Husserl acknowledges that further investigations are needed for "a clarification of the so-little understood relationship between 'descriptive' and 'explanatory' science" (Husserl 1982, 165). This field of phenomenological research belongs to a more general issue concerning the complex relationship between phenomenology and ontology. Although shedding light upon Husserl's complex view of this issue goes beyond the scope of the present paper[31], we would like to point out that a connection between a descriptive analysis of a field of inquiry and its exact determination can be established via an idealizing procedure intuitively ascertained. Such a connection is important if we want to provide an exact determination of that very field of inquiry, or we can say, of that *regional ontology*. We should interpret Weyl's investigations within Husserl's phenomenological framework.

Weyl's research on the nature of intuitive space aims to uncover the structure of space that underlies the domains of objects immediately given in our experience of space. Whereas "in examining a real object we have to rely continually on our sense perception in order to bring to light ever new features, capable of *description in concepts of vague extent only*", the structure of space "can be exhaustively characterized with the help of a few *exact concepts* and in a few statements, the *axioms*, in such a manner that all geometrical concepts can be defined in terms of those basic concepts and every true geometrical statement follows as a logical consequence from the axioms" (Weyl 1949, 3, my emphasis). Once intuition has "furnished us with the necessary basis", we shall "enter into *the region of deductive mathematics*" (Weyl 1952, 16, my emphasis). In this sense geometry turns out to be a "*theory of space*" (Weyl 1949, 18, my emphasis). Moreover, "the scientific theory in question is said to be *definite* (*definit*) according to Husserl" (Weyl 1949, 18).[32] Weyl's preference for the axiomatic construction of affine geometry over Euclid and Hilbert's approach can be also better understood within Husserl's phenomenological framework. Indeed, only the former theoretical construction takes into account the idealizing procedure involved in the constitution of the ideal essences of line and plane.[33] Moreover, the meanings of the exact concepts that express these ideal essences are better clarified within the phenomenal domain of intuitive space. Affine geometry, therefore, reveals "a wonderful harmony between the given on one hand and reason on the other" because it reflects the descriptive analysis of this field of inquiry more accurately (Weyl 1949, 69).

Similarly, Weyl's mathematical conceptualizations of the continuum find their roots in Husserl's phenomenological framework. In *Das Kontinuum*, Weyl tries to establish a connection between something given in the "*morphological description* of what presents itself in intuition" and "something constructed in a logical conceptual way" (Weyl 1994, 49, my emphasis). Nevertheless, any idealizing procedure can be intuitively ascertained as regards the constitution of the category of point. His research in topology improves this approach by developing a theoretical construction that takes into account the idealizing procedure involved in the constitution of the ideal essence of a point. This ideal essence is then expressed by an exact concept, whose meaning can be clarified within the phenomenal domain of intuitive continuum. Weyl further claims that, in order to improve this approach, the process of division itself should not be regarded as given in an exact way. In fact, we should assume that the divisions are given only vaguely and are not accurately done since an exact division would contradict the essence of the continuum. However, as the division progresses, the accuracy will increase indefinitely.[34] Topological studies allow us to exactly address these problems "even though the continua to which they are addressed may not be given exactly but only vaguely, as is always the case in reality" (Weyl 1949, 90). These studies provide an intermediate level of analysis since a rational analysis of continua "proceeds in three steps: (1) *morphology*, which operates with *vaguely circumscribed types of forms*; (2) *topology*, which, *guided by conspicuous singularities* or even in free construction, places into the manifold a vaguely localized but combinatorially exactly determined skeleton; and (3) *geometry* proper, whose *ideal structures* could only be carried with exactness into a real continuum after this has been spun over with a subdivision net of a fineness increasing *ad* *infinitum*" (Weyl 1949, 91, my emphasis). Husserl's influence on these topological studies is highlighted by Weyl's reference to O. Becker (1889-1964), who wrote a *Habilitationsschrift* in 1922 on the phenomenological foundations of geometry and relativity theory under Husserl's direction. For a "more careful phenomenological analysis of the *contrast between vagueness and exactness and of the limit concept*, the reader may be referred to the work by O. Becker", his *Beiträge zur phänommenologischen Begründung der Geometrie und ihrer physikalischen Anwendungen* (Weyl 1949, 91, my emphasis; cf. Becker 1923). Becker indeed further develops this analysis by improving especially the foundational aspects involved in the connection between a descriptive analysis of a field of inquiry and its exact determination.[35]

4. Conclusion

Weyl's research turns out to be an attempt to establish a connection between a descriptive analysis of phenomena and their exact determination within Husserl's phenomenological framework we have outlined. He tries to untangle this connection in different ways, and, for this reason, we should not come to the conclusion that Weyl keeps changing perspective in his studies between 1917 and 1927. Instead, his theories should be read as different attempts to attain a theoretical construction that is as much phenomenologically grounded as possible. In this sense, we can refer to Weyl's *phenomenological constructivism*. I would like to specify that I am not arguing that Weyl's studies can be defined exactly as phenomenological research. Weyl himself acknowledges that he broaches only "lightly on the philosophical implications" since he is not "in a position to give such answers to the epistemological questions involved" as his conscience would allow him to uphold (Weyl 1952, 2). In *Das Kontinuum*, for instance, he remarks that his research on the continuum is "only a slightly illuminating surrogate for a genuine philosophy of the continuum" since his task "is mathematical rather than epistemological" (Weyl 1994, 97). He also admits that for him it is "very difficult to give a precise analysis of the relevant mental acts" (Weyl 1995, 454).

Therefore, my interpretation suggests that Weyl's studies are developed by taking into account Husserl's phenomenological framework though they should be further clarified through an in-depth phenomenological analysis.

These considerations could be extended beyond the few examples we have shown. In particular, Weyl's development of *infinitesimal geometry* should be understood within this framework. In *Raum-Zeit-Materie*, Weyl addresses the rising theory of general relativity and aims to develop a theoretical construction of the real world whose meaning is phenomenologically clarified within the domain of our experience.[36] To conclude, here are the philosophical reasons that underlie Weyl's famous remark in *Raum-Zeit-Materie*:

> The investigations about space that have been conducted in chapter II appear to me to offer a good example of the essential analysis (*Wesenanalyse*) striven for by phenomenological philosophy (Husserl), an example that is typical for such cases where a non-immanent essence is dealt with (Weyl 1921a, 133, my translation).

NOTES

[1] For a general introduction to Weyl's scientific and philosophical work see Scholz (2001) and Bell and Korté (2016).
[2] These studies, of course, shed light on many further details. For instance, in Da Silva (1997), Weyl's predictivism is clarified by reference to Husserl's theory of meaning proposed in *Logische Untersuchungen*.
[3] In Weyl's obituary, appeared in the *Biographical Memoirs of Fellows of the Royal Society* in 1957, the years between 1917 and 1927 are described as the period when Weyl "was at the height of his powers" (Newman 1957, 306). It was a rich and stimulating period for Weyl's mathematical and philosophical production and a substantial body of work was published at that time. For this reason, the decade 1917-1927 is an important period to focus on.
[4] *Philosophy of Mathematics and Natural Science* (1949) is the revised English version of the first German edition published in 1927 (Weyl 1927). The text was translated by O. Helmer with the help of J. Weyl, and it was reviewed by Hermann Weyl himself. There are no significant changes from the first edition, except for six essays that Weyl added. For these reasons, in most cases I will refer to the English edition, while I will quote directly from the German edition when needed.
[5] We are considering properties among relations as a special case.
[6] On the contrary, a proposition is meaningless (*sinnlos*) when this condition is not satisfied. For instance, the judgment scheme "H(x): x is honest" does not yield a meaningful proposition if x is filled by the object "leaf". Weyl's theory of meaning is thoroughly described in Tieszen (2000).

[7] Actually Weyl seems to believe that a proper intuitive analysis of the given field of inquiry would prevent us from being trapped in such absurdities. He says: "Perhaps meaningless propositions can appear only in thought about language, never in thought about things" (Weyl 1994, 5).
[8] This quotation is taken from an article published in 1919 (Weyl 1919). It has been translated in English and added as an appendix in Weyl (1994).
[9] Weyl's principles of logical construction belong to a more comprehensive "logical critique of language" (Weyl 1949, 7). Specifically, he speaks of *pure grammar* when referring to Husserl's *Logische Untersuchungen*. See Weyl (1994, 113, note 2). He makes reference to Husserl's philosophy of logic also in the preface of *Das Kontinuum* when he says: "Concerning the epistemological side of logic, I agree with the conceptions which underlie Husserl's *Logische Untersuchungen*. The reader should also consult the deepened presentation in Husserl's *Ideen* which places the logical within the framework of a comprehensive philosophy" (Weyl 1994, 2).
[10] A similar remark can be found in Weyl's *infinitesimal geometry*. This more general approach improves our analysis of space to such an extent that Weyl refers to it as "the climax of a wonderful sequence of logically-connected ideas, and in which the result of these ideas has found its ultimate shape, is a true *geometry*, a doctrine of *space itself*" (Weyl 1952, 102).
[11] For a detailed account of the notion of manifold (*Mannigfaltigkeit*) from the mid-nineteenth century to the mid-twentieth century, see Scholz (1999). The historical development of this concept is complex and it is not always easy to recognize the meaning assigned by each author. For a better understanding of Weyl's notion of manifold, however, we can notice what he says with regard to the notion of *surface* in *Die Idee der Riemannschen Fläche*: "[…] the concept 'two-dimensional manifold' or 'surface' will not be associated with points in three-dimensional space; rather it will be a much more general abstract idea. If any set of objects (which will play the role of points) is given and a continuous coherence between them, similar to that in the plane, is defined, then we shall speak of a two-dimensional manifold" (Weyl 2009, 16). Therefore, Weyl's notion of manifold can be broadly understood as the "abstract form" of a given field of inquiry.
[12] He further adds that a point O and three linearly independent vectors constitute a *coordinate system*. This system allows us to identify a point by its *coordinates* $\lambda_1, \lambda_2, \lambda_3$ by means of the relation $\overrightarrow{OP} = \lambda_1\overrightarrow{e_1} + \lambda_2\overrightarrow{e_2} + \lambda_3\overrightarrow{e_3}$.
[13] In *Das Kontinuuum* Weyl remarks that: "[…] the *continuity* given to us immediately by intuition (in the flow of time and in motion) has yet to be grasped mathematically as a totality of discrete "stages" in accordance with that part of its content which can be conceptualized in an "exact" way. More or less arbitrarily axiomatized systems (be they ever so "elegant" and "fruitful") cannot further help us here" (Weyl 1994, 24).
[14] Weyl follows implicitly Husserl's approach. Husserl argues that a preliminary act is needed in the analysis of experience. He refers to it as *epoché* and it is conceived as the suspension of judgment about the natural world, setting aside all objective theses and focusing on the phenomenon as it presents itself. See Husserl (1913b).

[15] This equality refers to the equality of experiential content of the two *time spans* AB and A'B', into which falls every time-point that is later than A(A'), but earlier than B(B'). Weyl actually remarks that such an equality might be very controversial, however, he chooses to not delve into it. As we will see, the previous postulate will turn out to be an even bigger issue.
[16] Weyl's construction of real numbers in *Das Kontinuum* is logically built on the basic category of natural numbers and the primitive relation "S(n',n): n' is the successor of n". He develops this construction in detail and introduces many other notions, such as the notions of *set* and *function*. For further details, see Mancosu (2010). For an axiomatic interpretation, see Feferman (1988).
[17] The idea of isomorphism then turns out to be "of fundamental importance for epistemology" (Weyl 1949, 25). Weyl also refers to *transfer principle* (*Übertragungsprinzips*). By adopting an isomorphic mapping between two domains "is possible to transfer any insights gained in one field to the isomorphic field" (Weyl 1949, 26). Similar considerations are also supported in the case of a mathematical theory of space. With regard to space Weyl states: "[…] for example, Descartes' construction of coordinates maps the space isomorphically into the operational domain of linear algebra" (Weyl 1949, 25). Weyl further claims that a mathematical theory of time or space cannot be pursued as an independent axiomatic science but it should rely on this transfer principle. We should transfer any result pertaining to analysis into the domain of time-points by means of "a transfer principle based on the introduction of a coordinate system" (Weyl 1994, 96). Weyl finally remarks that the notion of isomorphism "induce us to conceive of an axiom system as a *logical mold* (*Leerform*) of possible sciences" (Weyl 1949, 25). A concrete interpretation is given "when designata have been exhibited for the names of the basic concepts, on the basis of which the axioms become true propositions" (Weyl 1949, 25). "Pure mathematics, in the modern view, amounts to a general hypothetico-deductive theory of relations; it develops the theory of logical 'mold' without binding itself to one or the other among the possible concrete interpretations" (Weyl 1949, 27). In line with Husserl, Weyl points out that the notion of *formalization* reflects a point of view "without which an understanding of mathematical methods is out of the question" and suggests the reader to "compare Husserl, *Logische Untersuchungen*, I, Section 67-72" (Weyl 1949, 27).
[18] Weyl inherits this conception of the real world from Husserl. He explicitly makes reference to Husserl's *Ideen* when he claims: "[…] the real world, and every one of its constituents with their accompanying characteristics, are, and can only be given as, intentional objects of acts of consciousness" (Weyl 1952, 4).
[19] Weyl recommends reading Husserl's phenomenological description of time (Husserl 1913b, § 81,82) for further details. He makes also reference to Bergson's philosophy (Bergson 1907).
[20] Weyl's studies are often characterized by a continuous tension between a temporary solution and a call for a better solution. For this reason, these considerations are not in conflict with previous mathematical conceptualizations of space. In this case, Weyl is showing us the underlying

problems concerning a finer analysis of a mathematical theory of time or space.

[21] "[…] sie dadurch gegen das Wesen des Kontinuums verstößt, als welches seiner Natur nach gar nicht in eine Menge einzelner Elemente zerschlagen werden kann. Nicht das Verhältnis von Element zur Menge, sondern dasjenige des Teiles zum Ganzen sollte der Analyse des Kontinuums zugrunde gelegt warden" (Weyl 1988, 5).

[22] Weyl is referring to Husserl's *Logische Untersuchungen*. See Husserl (1973, vol II, 29).

[23] Note that he did not always agree with Brouwer and that he actually put forward his own foundational account. For a comparison between them, see van Atten *et al*. (2002).

[24] The picture and the following remarks are taken from another paper published in 1940 and titled *The Mathematical Way of Thinking* (Weyl 1940). In that paper, this account is better explained.

[25] Weyl outlines how we can develop a mathematical analysis of this manifold in accordance with his previous foundational remarks. However, he was aware that several issues should have been addressed, and his research on *combinatorial topology* aims to further develop this approach. He published two important contributions in that direction in 1923 and 1924. See Weyl (1923, 1924) and Scholz (2000).

[26] See Weyl (1948, 381). For further details about the personal contacts between Weyl and Husserl, see Ryckman (2005, § 5).

[27] The correspondence is published in Schuhmann (1996) and in van Dalen (1984). Few excerpts are translated and discussed in Ryckman (2005, § 5). See also Tonietti (1988). For a French translation that also includes a noteworthy letter from Weyl to Husserl, see Lobo (2009).

[28] Husserl's notion of *definiteness* (*Definitheit*) has been a matter of debate, especially in relation to the modern notion of *completeness*. A number of different interpretations of this notion have been proposed in the literature. See, for instance, Ortiz Hill (1995), Majer (1997), Da Silva (2000) and Centrone (2010). For a detailed account of the various notions of completeness that were theorized in connection with the development of the axiomatic method in the late nineteenth and early twentieth century mathematics, see Awodey and Reck (2002).

[29] Husserl points out, however, that phenomenology is not an inadequate science because it is not an exact science. Our prejudices on the well-known exact sciences, such as geometry, should not make us fail to recognize that "transcendental phenomenology, as a descriptive science of essence, belongs however to a *fundamental class of eidetic sciences totally different* from the one to which the mathematical sciences belong" (Husserl 1982, 169).

[30] Husserl says: "In the eidetic province of reduced phenomena (either as a whole or in some partial province) […] the pressing question of whether, besides the descriptive procedure, one might not follow - as a counterpart to descriptive phenomenology - an idealizing procedure which substitutes pure and strict ideals for intuited data and might even serve as the fundamental means for a mathesis of mental processes" (Husserl 1982, 169).

[31] For further details, see Husserl (1982, § 72-75) and Husserl (1980, § 13-17). In later years, Husserl revises his analysis of idealization as part of his historical reflection on the origins of philosophical and scientific thought. Important remarks concerning the origin of geometry can be found in his *Krisis*. See Husserl (1970, 353).

[32] For a better understanding of Weyl's notion of definiteness note that Weyl distinguishes it from the notion of *completeness* (*Vollständigkeit*). He claims that, in a complete system of axioms, for every pertinent general proposition a the question 'does a or ~a hold?' could be answered by using logical inference on the basis of the axioms, however, "mathematics would thereby be trivialized" (Weyl 1949, 24). Intuition and "the life of the scientific mind pose the problem, and these cannot be solved by mechanical rules like computing exercise" (Weyl 1949, 24). Cf. Centrone (2010, § 3.6.2) for a comparison between Husserl's notion of *Definitheit* and Hilbert's notion of *Vollständigkeit*.

[33] Note that affine geometry does not take into account any idealizing procedure regarding the basic categories of objects. Line and plane are indeed "derived exact concepts". However, the connection between a descriptive analysis of a field of inquiry and its exact determination should be established, wherever appropriate.

[34] "In der Wirklichkeit muß man sich vorstellen, daß die Teilung auf der 0^{ten} Stufe Σ_0 [on S] nur vage, mit einer beschränkten Genauigkeit gegeben ist; denn eine exakte Teilung widerspricht dem Wesen des Kontinuums. Aber bei fortschreitender Teilung soll sich auch die Genauigkeit, mit der die anfänglichen Ecken und Seiten und die auf den vorhergehenden Stufen neu eingeführten festgelegt sind, unbegrenzt steigern" (Weyl 1988, 8).

[35] In a letter to Weyl dated April 9, 1922, Husserl wrote: "Dr Becker also found it necessary in the first part of his work to enter into the general fundamental questions concerning the theorization of vague experiential data, with its vague continuity, and to sketch a constitutive theory of the continuum" (Mancosu and Ryckman 2010a, 282). For further details, consult the correspondence between Weyl and Becker, which is discussed in Mancosu and Ryckman (2010a, 2010b). See also Lobo (2009).

[36] In a letter to Weyl dated April 12, 1923, Becker remarks that Weyl's work on general relativity has for the first time "made possible a complete phenomenological foundation for geometry (in the sense of 'world geometry')". He further adds that "the same idealistic conception" underlies both Weyl's theory of continuum and his infinitesimal geometry. See Mancosu and Ryckman (2010b, 309).

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